MATH 350: Graph Theory and Combinatorics. Fall 2016.
Assignment #2: Bipartite graphs, Matchings, Connectivity

Due Wednesday, October 19th, 2016, 14:30

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1. Recall that a graph G is called *d-regular* if every vertex of G has degree equal to d.

a) Construct a 3-regular graph that does not contain a perfect matching. You have to prove that the constructed graph does not contain a perfect matching.
Hey, I bet your graph doesn't contain a 2-factor either! A coincidence?

b) Prove the following statement: Let G be a 3-regular graph. G contains a perfect matching $\iff$ G contains a 2-factor.
. . . Ah, so no coincidences on this sheet!

2. Prove that every graph $G = (V, E)$ contains a subgraph H that is bipartite and has at least $|E|/2$ edges.

3. Let G be a connected graph. We say that $F \subseteq E(G)$ is *even-degree*, if every vertex of G is incident with an even number of edges in F. Let T be an arbitrary spanning tree of G. Prove that there exists an even-degree set $F_T \subseteq E(G)$ such that $F_T \cup E(T) = E(G)$.

[*Hint: First, prove that if two sets F and F' are both even-degree, then so is the set* $F \triangle F' := (F \setminus F') \cup (F' \setminus F)$.]

4. Let G be a 3-regular graph. Show that the edge connectivity $\kappa_e(G)$ is equal to the vertex connectivity $\kappa_v(G)$.

5. Let G be an n-vertex bipartite graph such that every degree of G is between 10 and 20. Show that G contains a matching of size at least $n/3$.