**Instructions:** The exam is 3 hours long and contains 6 questions. The total number of points is 100. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. Justify all your answers!

**Q1** Let $G$ be the graph depicted in Figure 1.
   
   a) Is $G$ planar? (4 points)
   b) Find $\nu(G)$ and $\tau(G)$. (4 points)
   c) Find $\chi(G)$. (4 points)
   d) Find $\chi'(G)$. (4 points)

**Q2** Let $\overrightarrow{G} = (V,E)$ be the oriented graph with the two specific vertices $s$ and $t$ and with the capacities $c : E \to \mathbb{Z}_+$ depicted in Figure 2.
   
   a) Find a maximum flow from the vertex $s$ to the vertex $t$. (8 points)
   b) Find a minimum $s,t$-cut. (8 points)

**Q3** Let $G = (V,E)$ be the simple graph with weights $w : E \to \mathbb{Z}_+$ obtained from the oriented graph depicted in Figure 2 by replacing each oriented edge by a non-oriented one that has the same weight.
   
   a) Find a minimum-cost spanning tree in $G$. (8 points)
   b) Does $G$ have a unique minimum-cost spanning tree. (8 points)

**Q4** Let $k \geq 1$ be an integer, and let $G$ be a connected $2k$-regular graph. Show that $G$ is 2-edge-connected. (17 points)

**Q5** Let $G$ be a simple planar graph. Prove that if $G$ contains no cycle of length five or less, then $\chi(G) \leq 3$. (17 points)

**Q6** Let $K_4^-$ be the 4-vertex graph obtained from $K_4$ by removing one edge. How many non-isomorphic simple 2-connected graphs $G = (V,E)$ are there with $|V| = 1000$ such that $G$ has no $K_4^-$-minor? (18 points)
Figure 1: The graph in the question Q1.

Figure 2: The oriented graph in the questions Q2 and Q3.