Instructions: The exam is 3 hours long and contains 6 questions. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. Justify all your answers.

Q1  Let $G$ be the graph depicted in Figure 1.
   a) Is $G$ planar?
   b) Find $\tau(G)$.
   c) Show that $\chi(G) = 4$.
      (Hint: To prove that $\chi(G) > 3$, show that in any 3-coloring of the graph $G \setminus v$, all three colors will appear on the neighborhood of $v$.)

Q2  Let $G$ be the graph with weights $w : E(G) \to \mathbb{Z}_+$ depicted in Figure 2.
   a) Find the min-cost spanning tree in $G$.
   b) Find a shortest path spanning tree for the vertex $A$.

Q3  Let $k \geq 1$ be an integer, and let $G$ be a connected $k$-regular bipartite graph. Show that $G$ is 2-connected.

Q4  Let $G$ be a planar graph. Show that if $G$ does not contain any cycles of length five or less then $\chi(G) \leq 3$.

Q5  Let $G$ be a 3-connected graph with $|V(G)| \geq 5$. Show that $C_5$ is a minor of $G$.

Q6  Let $G$ be a simple graph. Suppose that there exists a clique $X \subseteq V(G)$ such that $V(G) - X$ is an independent set. Show that $G$ is perfect without using the strong perfect graph theorem.
Figure 1: The graph in the question Q1.

Figure 2: The graph in the question Q2.