Instructions: The exam is 3 hours long and contains 6 questions. The total number of points is 100. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers!**

Q1 Let G be the graph depicted in Figure 1.

- **a)** Is G planar? *(4 points)*
- **b)** Find $\nu(G)$ and $\tau(G)$. *(4 points)*
- **c)** Find $\chi(G)$. *(4 points)*
- **d)** Find $\chi'(G)$. *(4 points)*

Q2 Let $\overrightarrow{G} = (V, E)$ be the oriented graph with the two specific vertices s and t and with the capicities $c : E \to \mathbb{Z}_+$ depicted in Figure 2.

- **a)** Find a maximum flow from the vertex S to the vertex T. *(8 points)*
- **b)** Find a minimum S, T-cut. *(8 points)*

Q3 Let $G = (V, E)$ be the simple graph with weights $w : E \to \mathbb{Z}_+$ obtained from the oriented graph depicted in Figure 2 by replacing each oriented edge by a non-oriented edge with the same weight.

- **a)** Find a shortest path spanning tree in G for the vertex S. *(6 points)*
- **b)** Find a minimum-cost spanning tree in G. *(6 points)*
- **c)** Does G have a unique minimum-cost spanning tree? *(6 points)*

Q4 Let $k \geq 2$ be an integer, and let G be a connected k-regular bipartite graph. Prove that G is 2-connected. *(16 points)*

Q5 Let G be a simple planar graph. Prove that if G contains no cycle of length five or less, then $\chi(G) \leq 3$. *(16 points)*

Q6 Let K_4^- be the 4-vertex graph obtained from K_4 by removing one edge. How many non-isomorphic simple 2-connected graphs $G = (V, E)$ are there with $|V| = 1000$ such that G has no K_4^--minor? *(18 points)*

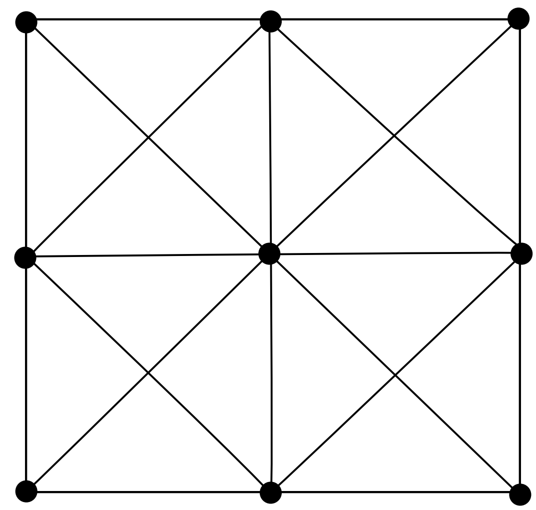

Figure 1: The graph in the question Q1.

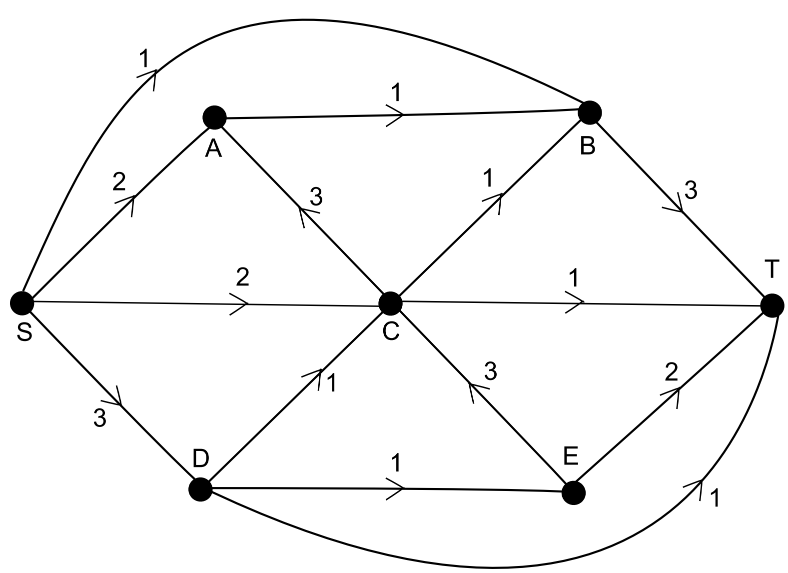

Figure 2: The oriented graph in the questions Q2 and Q3.