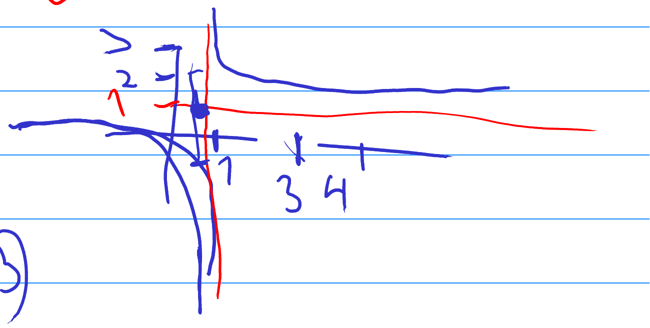
Cvičení začíná 7:35

2.4 $\neg(A \wedge B) \iff (\neg A \vee \neg B)$ — L, P

ROZBOR PŘÍPADŮ (4 možnosti)

A	B	L	P
0	0	1	1
0	1	1	1
1	0	1	1
1	1	∅	∅

2.30 $f:\mathbb{R}\to\mathbb{R}$ $f(x) = x+1/x-1$

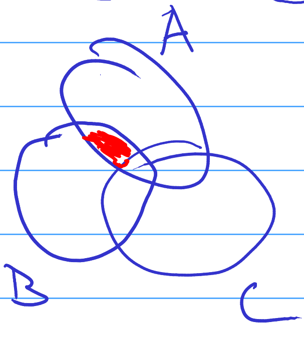


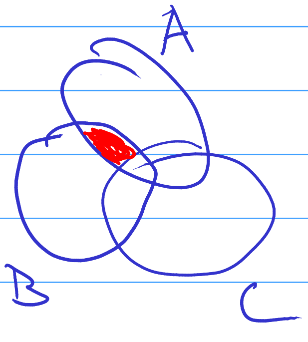
(ii) $f^{-1}(M)$ $M=(2,3)$

$f^{-1}(3)=2$ $f^{-1}(2)=3$ $f^{-1}(M)=(2,3)$

3.2 f^{-1} z $f(x)=\frac{ax+b}{cx+d}$

3.14 $L:=(A\setminus C)\cap B = A\cap(B\setminus C)=:P$





$L\subseteq P$ $P\subseteq L$

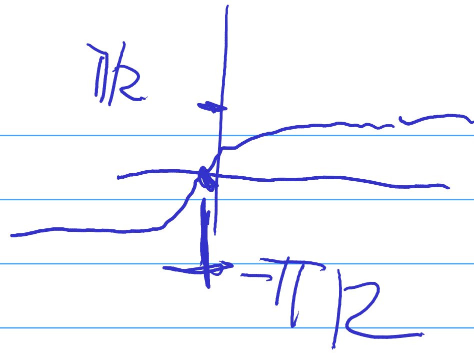
$\neg\ \exists x\in L\setminus P$ $\neg\ \exists y\in P\setminus L$

$x\in B$, $x\in A$, $x\notin C \Rightarrow x\in P$ ↯

$y\in A$, $y\in B$, $y\notin C \Rightarrow y\in L$ ↯

3.10 i)

$\sin(\operatorname{arctg}(x))$

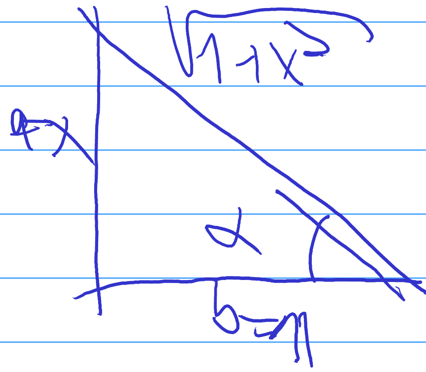


NÁVRH: zkusme $\sin(z)$ vyjádřit pomocí tg

$z \in (-\pi/2, \pi/2)$

$1 = \sin^2 x + \cos^2 x$

$\sin \cancel{x} z = \operatorname{tg}(z) \cdot \cos(z)$



$\operatorname{tg}\alpha = a/b$

$\operatorname{arctg}\left(\frac{a}{b}\right) = \alpha$

$\sin(\alpha) = \frac{x}{\sqrt{1+x^2}}$

$z \in (-\pi/2, \pi/2)$

$\operatorname{tg} z = \frac{\sin z}{\cos z} \rightsquigarrow \operatorname{tg}^2 z = \frac{\sin^2 z}{1-\sin^2 z}$

$\operatorname{tg}^2 z = \sin^2 z + \sin^2 z \operatorname{tg}^2 z$

$\sin^2 z = \frac{\operatorname{tg}^2 z}{1+\operatorname{tg}^2 z} \rightsquigarrow \sqrt{\sin^2(\operatorname{arctg} x)} = \sqrt{\frac{x^2}{1+x^2}}$

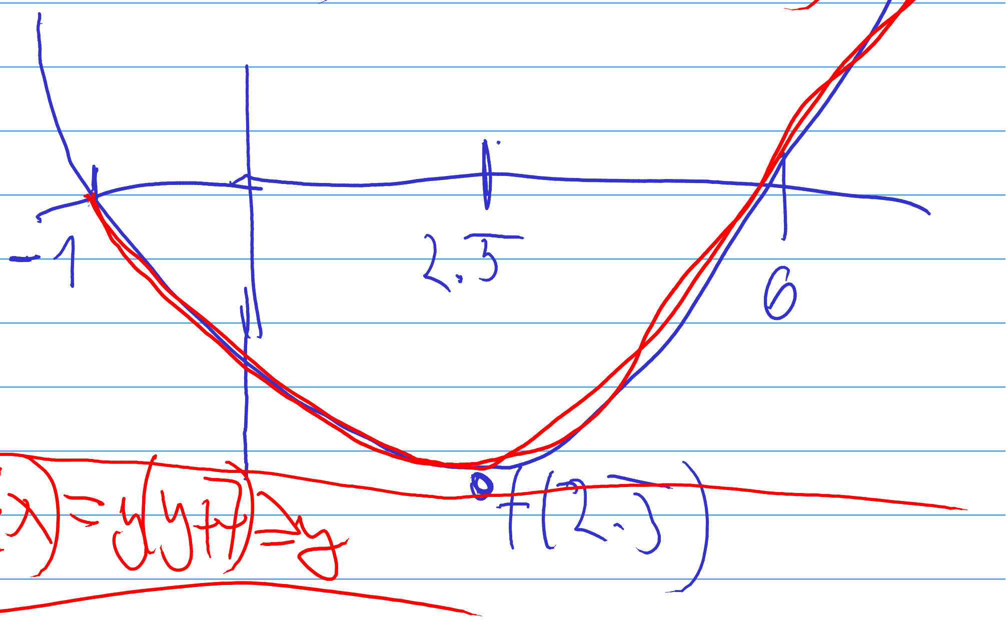
$\Rightarrow |\sin(\operatorname{arctg} x)| = \frac{|x|}{\sqrt{1+x^2}}$ z |o| na L a P $\Rightarrow$

$\sin(\operatorname{arctg} x) = x/\sqrt{1+x^2}$

3.23) $M = \{x^2 - 5x - 6,\ x \in (-1, \infty)\} \subseteq \mathbb{R}$

je omezená? NEomezená shora a omezená zdola

$(x-6)(x+1)$ $\geq f(2.5)$



$\forall y \geq 0$

$x := y + 6$ $\quad f(x) = y(y+7) \geq y$

$f(x) = \underbrace{\alpha(x-\beta)^2}_{\geq 0} + \gamma$ $\quad \forall x$

$f(x) \geq \gamma$

$\alpha > 0$ $\quad \alpha = 1$

$\gamma = f(2.5)$

$\beta = 2.5$

{

Pokud M má maximum y
$\Rightarrow$ M má supremum y
pokud M má minimum x
$\Rightarrow$ M má infimum x

$N = \{x \in \mathbb{Q},\ x^2 < 2\}$
$\sup N = \sqrt{2}$
$\inf N = -\sqrt{2}$
nemá max / min

3.24

jsou zdola/zhora omezené

(i) $\emptyset$ je omezená shora i zdola $\forall x \in \mathbb{R}$

M je omezená shora $\equiv \exists x \in \mathbb{R}$

$\forall y \in M: y \leq x$

$\forall y \in \emptyset: y \leq x$

$\forall y \in \emptyset$: ~~~~~~

$\forall x \in X$ $\underbrace{\text{podminka}}$: $V(x)$

$X = \emptyset$ PRAVDA

DEF

3.24 (ii)

$\left\{ \frac{2n}{3n+7} \;\middle|\; n \in \mathbb{N}_{\geq 0} \right\} =: M$

$\{a_n\}$

$a_n := 2n/3n+7$

M zdola omezená $\Leftarrow a_1 < a_n \; \forall n \geq 2$

$$\frac{2n}{3n+7}(3n+4) < \frac{(2n+2)}{3n+4}(3n+7)$$

a_n a_{n+1}

$\lim a_n = 2/3$

$\forall n \in \mathbb{N}$

$$\frac{2n}{3n+7} < 2/3 \quad ?$$

$$6n < 6n+7 \quad \checkmark$$

$\sup\{a_n\} = 2/3$ max $\{a_n\}$ нема!

$\inf(\{a_n\}) = \min(\{a_n\}) = 1/2$

$$H = [2/3, \infty)$$

$$D = (-\infty, 1/2]$$

$$\{1/2 < 4/7 < 6/10, \ a_n > a_{n+1}\}$$

$\leq 2/3$

$$n = 42^{42^{42}}$$

$f(x) = \frac{2x}{3x+7}$ $M' \supseteq M$

$\sup M' = \sup M$

$\min M' = \min M$

$M' := f([1, \infty))$

$n \geq 42^{42}$

$a_n \geq (42)^2$

$$f(x) = \frac{2}{3} - \frac{2/3}{3x+7} = \frac{2}{3} - \frac{2}{9x+13}$$

$a_n \leq 2/3$

$a_n = \frac{2}{3} - \boxed{\frac{2}{9n+13}}$ ↘ $\Rightarrow$ a_n горбне!

$a_1 = \min$