Start 7:35

4.10

$$\inf \left\{ \frac{3x+1-2x^2}{x^2+5x} \,\middle|\, x \in \mathbb{R}^+ \right\} = -2$$

NE

$\inf M \in D(M) \leftarrow$ dolní závory M

-2 je dolní závora

$$\# \quad \frac{3x+1-2x^2}{x^2+5x} \geq -2 \quad \forall x \in \mathbb{R}^+$$

$$3x+1-2x^2 \geq -2x^2 - 10x$$

$$13x \geq -1 \quad \checkmark \quad -2 \text{ je dolní závora}$$

$\forall \varepsilon > 0 \ \exists x_1 \ \# < -2 + \varepsilon$

$$\lim \frac{13n+1}{n^2+5n} \to 0$$

$$\frac{3x+1-2x^2}{x^2+5x} + 2 < \varepsilon \iff \frac{13x+1}{x^2+5x} < \varepsilon$$

$$\frac{13x+1}{x(x+5)} \leq \frac{13x+5\cdot 13}{x(x+5)} = \frac{13}{x} < \varepsilon$$

$$x := 13/\varepsilon$$

$$\frac{13x+1}{x(x+5)} + \frac{64}{x(x+5)} = \frac{13\cancel{(x+5)}}{x\cancel{(x+5)}} = 13/x$$

4.12

$$\sup \left\{ \frac{n^2 + 3n + 5}{1 - 2n} \;\middle|\; n \in \mathbb{N} \right\}$$

$$\boxed{f(x) = \frac{x^2 + 3x + 5}{1 - 2x}} \qquad \sup = \frac{9}{5}$$

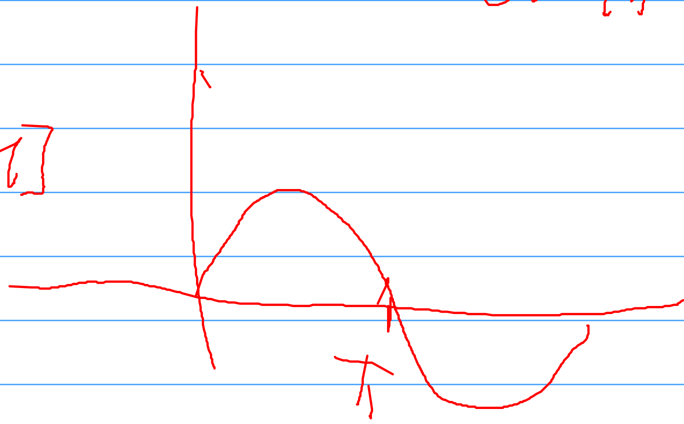
extrémy $f(x)$ $\qquad x = \frac{1 + 3\sqrt{5}}{2}$

$\lfloor x \rfloor , \lceil x \rceil$

4.15 $\inf\{\sin\frac{1}{n}, n \in \mathbb{N}\}$

$\sin 1, \sin 1/2, \ldots \sin 1/n \ldots$

$\{1/n : n \in \mathbb{N}\} \subseteq [0, 1]$



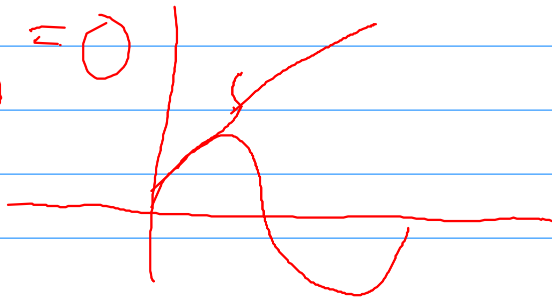
sin je klasijični

$1/n$ je klasijični $\Rightarrow \inf \sin(1/n) = \lim_{n\to\infty} \sin(1/n)$

$\lim_n \sin(1/n) = \sin(\lim_n 1/n) = 0$

$\sin x \leq x \quad \forall x \geq 0$

$\lim \sin(1/n) \leq \lim 1/n = 0$



4.22 (i) $a_n^{\to 0} = \dfrac{2n+3}{n^2+3n+1}$ $\lim a_n = 0$

monotóní? omezená? ANO

$\left(\underbrace{\frac{5}{5}, \frac{7}{11}, \frac{9}{19}}, \dots\right)$ $\max\{a_n\} = a_1 = 1$

$\forall n \in \mathbb{N}: a_{n+1} < a_n$

$$\frac{2n+5}{n^2+5n+5} < \frac{2n+3}{n^2+3n+1}$$

$$(2n+5)(n^2+3n+1) < (2n+3)(n^2+5n+5)$$

$$\cancel{2n^3} + 6n^2 + 2n + 5n^2 + 15n + 5 < \cancel{2n^3} + 10n^2 + 10n + 3n^2 + 15n + 15$$

$$11n^2 + 17n + 5 < 13n^2 + 25n + 15$$

$$0 < 2n^2 + 8n + 10$$

4.22. (ii) $a_n = (n-\sqrt{n})^n \qquad n \in \mathbb{N}$

$a_n \geq 0 \qquad a_1 = 0$

$a_2 = (2-\sqrt{2})^2 > 0$

$n - \sqrt{n}$ rostoucí

$\quad \| $

$[\sqrt{n} \cdot (\sqrt{n}-1)]$

$f(g(x))$ je rostoucí ⟸ $f(\cdot)$ rostoucí, g rostoucí ?

$a_{n+1} > a_n$

$\boxed{[\underbrace{\sqrt{n+1}}_{>0}\,(\underbrace{\sqrt{n+1}-1}_{>0\ n\geq 1})]^{n+1}} > [\sqrt{n+1}(\sqrt{n+1}-1)]^n \overset{?}{\geq} \boxed{[\sqrt{n}(\sqrt{n}-1)]^n}$

$\Uparrow$

$\sqrt{n+1}\,(\underbrace{\sqrt{n+1}}_{\substack{\vee \\ \sqrt{n}}}-1) > \sqrt{n+1}(\sqrt{n}-1) \overset{?}{\geq} \sqrt{n}(\sqrt{n}-1)$

$\Uparrow$

$\boxed{\sqrt{n+1} \geq \sqrt{n}}$

(a_n) je ostře rostoucí, speciálně $\min\{a_n\} = a_1 = 0$

$\lim\limits_{n\to\infty} a_n = \lim\limits_{n\to\infty} [\sqrt{n}(\sqrt{n}-1)]^n \geq \lim\limits_{n\to\infty} \sqrt{n}(\sqrt{n}-1)$

$= \lim\limits_{n\to\infty} \sqrt{n} \cdot \lim\limits_{n\to\infty} \sqrt{n}-1 = \infty \cdot \infty = \infty$

(a_n) NENÍ omezená shora

4.19: $A \subseteq \mathbb{R}$ $\qquad -A := \{-x : x \in A\}$

$s := \sup(-A) = -\inf(A) =: i$

CHCI $\qquad$ UI'M

① $\forall y \in -A: \ s \geq y$

② $\forall \varepsilon > 0 \ \exists y_\varepsilon \in -A: \ s - \varepsilon < y_\varepsilon$

$\forall y \in -A \ s \geq y$

$\forall x \in A: -i \geq -x \iff$ ① $\forall x \in A: \ i \leq x$

② $\forall \varepsilon > 0 \ \exists x_\varepsilon \in A: \ i + \varepsilon > x_\varepsilon$

$\forall \varepsilon > 0 \ \exists x_\varepsilon \in A: \ -i - \varepsilon < -x_\varepsilon$

$\forall \varepsilon > 0 \ \exists y_\varepsilon \in -A: \ s - \varepsilon < y_\varepsilon$

① $\iff$ ①

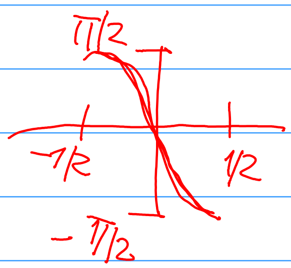
② $\iff$ ②

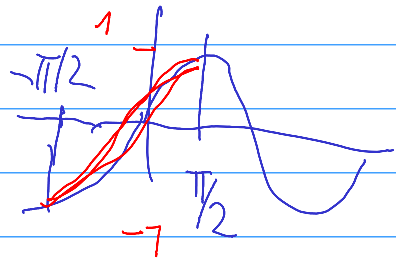
$-\sin(x)/2$

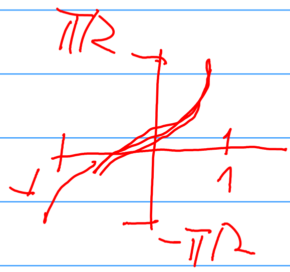
$g(x) := \sin(-x)/2 \rightsquigarrow g(\pi/2) = \sin(-\pi) = 0$

$f(x) := \arcsin(-2x)$

$f(-1/2) = \pi/2$







$\arcsin(-\cancel{2} \cdot \sin(-x)/\cancel{2})$ $\qquad \sin(-x) = -\sin(x)$

$= \arcsin(-\sin(-x)) = \arcsin(\sin x) = x$