START 7:37

5.1 $k \in \mathbb{Q}$: $\lim_{n\to\infty} n^k$

$k<0$ $\quad$ $k=0$ $\quad$ $k>0$

$\lim_{n\to\infty} 1 = 1$

$\lim_{n\to\infty} \boxed{\frac{1}{n^k}} = \frac{1}{\lim_{n\to\infty} n^{-k}} = 0$

$k \in \mathbb{Q}^+$

$\lim_{n\to\infty} n^k = +\infty = \left(\lim_{n\to\infty} n\right)^k = (\infty)^{+k} = +\infty$

$(+\infty)^{1/2} = \sqrt{\infty} = +\infty$

Definitoricky $\forall k>0$: $(+\infty)^k = +\infty$

$0^0 = \cancel{1}$ NEDEFINOVANO V ANALÝZE

5.4

$\boxed{\lim_{n\to\infty} \log_{\frac{1}{2}} n} = -\lim_{n\to\infty} \boxed{\log_2(n)} = -\infty$

z t. ž. $\left(\frac{1}{2}\right)^z = n$ $\quad$ z t. ž. $2^z = n$

$2^{-z} = n$

$\forall K \; \exists n_0 = 2^{K+1}$ t. ž. $\log n > K \quad \forall n \geq n_0$

$$\lim \log_{\frac{1}{2}}(n) = -\infty$$

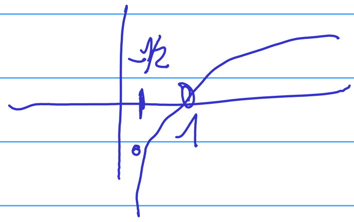
$$\Leftrightarrow \forall K>0 \; \exists n_0 \; \forall n \geq n_0 : \log_{\frac{1}{2}}(n) < -K$$

$$n_0 := \left(\frac{1}{2}\right)^{-(K+1)} = 2^{(K+1)}$$

$\forall n \geq n_0$

$$\log_{\frac{1}{2}} n \leq \log_{\frac{1}{2}} n_0 = -(K+1) < -K$$

$$\log_{\frac{1}{2}} n = \frac{\boxed{\log_e n}}{\log_e \frac{1}{2}} = \frac{\infty}{<0} = -\infty$$

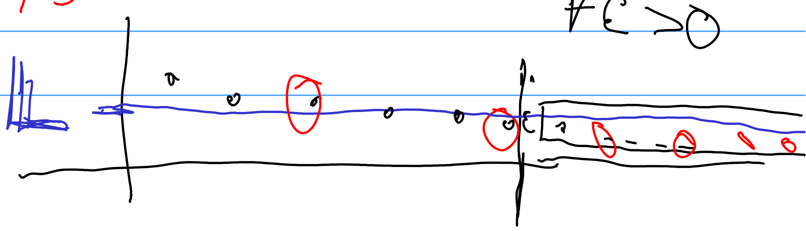


$$\Rightarrow \lim \log_z n \begin{cases} +\infty & z > 1 \\ -\infty & 0 < z < 1 \end{cases}$$

5.10 $a_n = \lim_{n\to\infty} \frac{(-1)^n}{2+3(-1)^n} \stackrel{?}{=} L$ $\quad \lim_{\ell\to\infty} a_{2\ell+1} = \frac{-1}{2-3} = 1$

$\lim_{k\to\infty}$ $\quad a_{2k} = \lim_{k\to\infty} \frac{1}{2+3} = 1/5$

$\forall \varepsilon > 0$



NOPE!

5.12.

$$\lim_{n\to\infty} \frac{n}{n+1}\cdot(-1)^{\frac{n(n+1)}{2}} \quad \text{NEEX.}$$

$$(a_{2k})$$

$$\lim_{k\to\infty} \frac{2k}{2k+1}\cdot(-1)^{k(2k+1)}$$

$$= \lim_{k\to\infty} \underbrace{\frac{2k}{2k+1}}_{\substack{\ge 2/3 \\ \le 1}} \left[(-1)\right]^{k} \le 1\cdot\underbrace{\lim(-1)^k}$$

$$\ge \underbrace{\tfrac{2}{3}\lim(-1)^k}$$

$$a_n \ge b_n$$

$$\underbrace{\lim_{n\to\infty} n\cdot(-1)^n} \ge \underbrace{\lim(-1)^n}$$

$$= +\infty$$

$$a_{2k} \to a_{4l}, \quad a_{4m+2}$$

$$\lim a_{4l} = \lim \frac{4l}{4l+1}\cdot \cancel{(-1)^{2l\cdot(4l+1)}}$$

$$= 1$$

$$\lim_{m\to\infty} a_{4m+2} = \lim_{m\to\infty} \frac{4m+2}{4m+3}\cdot\underbrace{(-1)^{\frac{4m+2}{2}\cdot(4m+3)}}_{}$$

$$(2m+1)(4m+3)$$

$$= \lim_{m\to\infty} -\frac{4m+2}{4m+3} = -1$$

5.17 (i) (a_n) omezená reálná
(b_n) reálná, $\lim_{n\to\infty} b_n = 0$

$$\lim_{n\to\infty}(a_n+b_n) \overset{?}{=} \lim_{n\to\infty} a_n$$

POKUD EXISTUJE

$a_n := (-1)^n$
PROČ $a_n + b_n$ nemá limitu?

$$+1 = \lim_{k\to\infty}(a_{2k}+b_{2k}) = \underbrace{\lim a_{2k}}_{\substack{\parallel \\ +1}} + \underbrace{\lim b_{2k}}_{+0}$$

$$-1 = \lim_{l\to\infty}(a_{2l+1}+b_{2l+1}) = LIM + LIM$$

$$\lim a'_n + b_n = \emptyset$$

$$a'_n := \emptyset$$

5.17 (ii) (a_n) omezená reálná

(b_n) $\lim\limits_{n\to\infty}(b_n) = \begin{cases} +\infty \\ \cancel{} \\ -\infty \end{cases}$

$\exists M \in \mathbb{R}\ \forall n\ -M \leq a_n \leq M$

$\forall n: \boxed{a_n + b_n} \leq \boxed{M + b_n}$

$\boxed{-M + \lim b_n} \leq \boxed{\lim\limits_{n\to\infty} a_n + b_n} \leq \lim\limits_{n\to\infty} M + b_n = M + \lim b_n$

$+\infty$ / $-\infty$; $+\infty$ / $-\infty$; $\downarrow$ $+\infty$ / $-\infty$

Věta:

$\exists n_0 \in \mathbb{N}\ \forall n \geq n_0: b_n \leq c_n \leq a_n$

$L = \lim b_n = \lim a_n \Rightarrow \lim c_n$ existuje & $L = \lim c_n$

$c_n := a_n + b_n$

$b_n \to +\infty$

$\lim\limits_{n\to\infty} b_n = +\infty$

TVRZENÍ

$\lim c_n = +\infty$

$\forall n: |a_n| \leq M \quad M \in \mathbb{R}$

Dk: $\forall k > 0: \exists n_0: \forall n \geq n_0 \quad c_n > k$

$H_k(\infty) = (\frac{1}{\varepsilon}, \infty)$

VÍME $\forall \ell > 0\ \exists n_1: \forall n \geq n_1 \quad b_n > \ell$

vějmeme $\ell = k + M \rightsquigarrow n_1(\ell)$ $\quad b_n > k + M \iff b_n - M > k$

$n_0(k) := n_1$

OVĚŘIT ŽE $\forall n \geq n_0 = n_1 \quad c_n > k$

$c_n = a_n + b_n \geq -M + b_n > k$

$\square$

5.26

$$\lim_{n\to\infty} \frac{(n+1)! - \boxed{(n-1)! \cdot n^2}}{n!} \qquad n! \times n$$

$$\parallel$$

$$\lim \frac{(n+1)\cancel{!}}{\cancel{n!}} - n = \lim (n+1) - n = 1$$

5.29 $a \in \mathbb{R}$:

$$\lim_{n\to\infty} \frac{(2n+a)^2}{(n-1)(3-5n)}$$

$$\lim \frac{4n^2 + 4n \cdot a + a^2}{-5n^2 + 8n - 3} \qquad \lim \frac{4}{-5 + 8/n - 3/n^2} = -4/5$$

$$= \boxed{\lim \frac{4n^2}{-5n^2+8n-3}} + \boxed{\lim \frac{4n \cdot a + a^2}{-5n^2+8n-3}} = 0$$

$$\forall a \qquad \lim_{n\to\infty} \boxed{\frac{4a}{n}} = 0$$

$$\lim_{n\to\infty} \boxed{\frac{4n \cdot a + a^2}{-5n^2+8n-3}} \qquad \to 0 \leftarrow$$

$\forall a \in \mathbb{R}\ \exists n_0 : \forall n \geq n_0$ $\quad \boxed{-4n^{3/2}} \leq 4n \cdot a + a^2 \leq \boxed{2n^{3/2}}$

$n \geq n_0 = a^2 \cdot 16$ $\qquad \forall n \geq n_0$:

$\Leftrightarrow a \leq \sqrt{n}/4$ $\qquad < \dfrac{n^{3/2} + n^{3/2}}{-5n^2+8n-3}$

$a \geq 0$ ✓ $\qquad a < 0$: $\quad n_0 = -a^2$

$a \geq -\sqrt{n}$

$$4n \cdot a + a^2 \geq 4n\,a \geq -4n^{3/2}$$

$$\lim \frac{c \cdot n^{3/2}}{-5n^2+8n-3} = \lim \frac{\boxed{c/\sqrt{n}}}{-5 + \boxed{8/n} - \boxed{3/n^2}} = 0/-5 = 0$$