START 7:40

6.25

$$\lim_{n\to\infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} = \lim_{n\to\infty} \frac{3^n\left(\left(-\frac{2}{3}\right)^n + 1\right)}{3^{n+1}\left(\left(-\frac{2}{3}\right)^{n+1} + 1\right)}$$

$$= \lim_{n\to\infty} \frac{1}{3} \cdot \left(\frac{\left(-\frac{2}{3}\right)^n + 1}{\left(-\frac{2}{3}\right)^{n+1} + 1}\right) = \frac{1}{3} \cdot \frac{\lim \left(-\frac{2}{3}\right)^n + 1}{\lim \left(-\frac{2}{3}\right)^{n+1} + 1} = 1/3$$

6.26

$a \in \mathbb{R}$

$$\lim_{n\to\infty} a^{n^2+n} \begin{cases} |a|>1 : +\infty \\ |a|=1 : 1 \\ a \in (-1,1) : 0 \end{cases}$$

$a>1$, 1) (a^{n^2+n}) je rostoucí, shora neomezená

2) je vybraná podposl. z (a^n)

$a=-1$ $\quad \lim_{n\to\infty} (-1)^{n^2+n} = \lim_{n\to\infty} (-1)^{n(n+1)} = 1$

$a<-1$: $\quad (a^{n^2+n}) = \left((a^2)^{\binom{n+1}{2}}\right)$ je vybraná z (a^n) $\quad \left(\frac{n(n+1)}{2}\right) = \binom{n+1}{2}$

$\binom{n+1}{2}$ – tý člen z $(a)^n$

$\emptyset$

$$n\left(\sqrt{1+\frac{1}{n}} + 2 - 3\sqrt{1+\frac{2}{n}}\right)$$

6.19

$$a - b \longrightarrow a + b$$

$$\left(\sqrt{n+1} + 2 - 3\sqrt{1+\tfrac{2}{n}}\right)$$

$$\lim_{n\to\infty} \sqrt{n}\left(\sqrt{n+1} + 2\sqrt{n} - 3\sqrt{n+2}\right)$$

$$= \lim \sqrt{n} \cdot \frac{\left(\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}\right)\left(\sqrt{n+1} + 2\sqrt{n} - 3\sqrt{n+2}\right)}{\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}}$$

$$= \lim \frac{\sqrt{n}\left(n+1 + 4n + 4\sqrt{n^2+n} - 9(n+2)\right)}{\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}}$$

$$-4n + 4n\sqrt{1+\tfrac{1}{n}}$$

$$4n\left(\sqrt{1+\tfrac{1}{n}} - 1\right)$$

$$\downarrow \infty \qquad \downarrow 0$$

$$= \lim \frac{\sqrt{n}\left(-17 - 4n + 4\sqrt{n^2\left(1+\frac{1}{n}\right)}\right)}{\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}}$$

$$= \lim \frac{16n(n+1) - \left(16n^2 + 136n + 289\right)}{\left(\sqrt{1+\frac{1}{n}} + 2 + 3\sqrt{1+\frac{2}{n}}\right)\left(4n + 17 + 4\sqrt{n(n+1)}\right)}$$

$$= \lim \frac{-120n - 289}{\left(\sqrt{1+\frac{1}{n}} + 2 + 3\sqrt{1+\frac{2}{n}}\right)\left(4n + 17 + 4\sqrt{n(n+1)}\right)}$$

$$= \lim \frac{-120 - 289/n}{\left(\sqrt{1+\frac{1}{n}} + 2 + 3\sqrt{1+\frac{2}{n}}\right)\left(4 + \frac{17}{n} + 4\sqrt{1+1/n}\right)}$$

$$= -120/48 = -5/2 \qquad \blacksquare$$

6.18

$$\lim \frac{1}{\sqrt{n}}\left(\sqrt{n+1}+\sqrt{2n}-\sqrt{3n+2}\right)$$

$$\lim \left(\sqrt{1+\frac{1}{n}}+\sqrt{2}-\sqrt{3+\frac{2}{n}}\right)$$

$$=1+\sqrt{2}-\sqrt{3}$$

6.15

$$\lim \sqrt[3]{n^3+n^2+1}-\sqrt[3]{n^3-n^2+1}$$

$$(a-b)\left(a^2+ab+b^2\right)=a^3-b^3$$

$$=\lim \frac{2n^2}{\left(n^3+n^2+1\right)^{2/3}+\left(\left(n^3+n^2+1\right)\left(n^3-n^2+1\right)\right)^{1/3}+\left(n^3-n^2+1\right)^{2/3}}$$

$$\lim 2/\left(\left(1+\frac{1}{n}+\frac{1}{n^3}\right)^{2/3}+\left(1-\frac{1}{n}+\frac{1}{n^3}\right)^{2/3}+\left(\left(1+\frac{1}{n}+\frac{1}{n^3}\right)\left(1-\frac{1}{n}+\frac{1}{n^3}\right)\right)^{1/3}\right)$$

$$=2/3$$

6.13 $\lim\limits_{n\to\infty} \sqrt{n^2+(-1)^n n+1} - \sqrt{n^2+(-1)^{n+1} n+1}$

$$\lim \frac{n\cdot\left((-1)^n + (-1)^{n\cancel{+1}}\right)}{\sqrt{n^2+(-1)^n n+1} + \sqrt{n^2+(-1)^{n+1} n+1}}$$

$$\lim 2(-1)^n \Big/ \left(\sqrt{1+\frac{(-1)^n}{n}+\frac{1}{n^2}} + \sqrt{1+\frac{(-1)^{n+1}}{n}+\frac{1}{n^2}}\right) :$$

(a_{2n}) $\quad \lim(a_{2n}) = 1$

(a_{2n+1}) $\quad \lim(a_{2n+1}) = -1$ $\Rightarrow \lim(a_n)$ neex!

$$\lim \frac{(-1)^n}{n} = 0$$

$$0 = \lim_{n\to\infty} -\frac{1}{n} \leq \lim \frac{(-1)^n}{n} \leq \lim \frac{\pm 1}{n} = 0$$

$$\lim \frac{2n^2}{(n^3+n^2+1)^{2/3}+\left((n^3+n^2+1)(n^3-n^2+1)\right)^{1/3}+(n^3-n^2+1)^{2/3}}$$

$$(n^3+n^2+1)^{2/3}=\left(n^3\left(1+\frac{1}{n}+\frac{1}{n^3}\right)\right)^{2/3}$$

$$= n^2\left(1+\frac{1}{n}+\frac{1}{n^3}\right)^{2/3}$$

$$\left[(n^3+n^2+1)(n^3-n^2+1)\right]^{1/3}=\left[n^6\left(1+\frac{1}{n}+\frac{1}{n^3}\right)\left(1-\frac{1}{n}+\frac{1}{n^3}\right)\right]^{1/3}$$

$$n^2\cdot\left(1+\ldots\right)^{1/3}\left(1-\ldots\right)^{1/3}$$