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6.25
$$\lim_{n \rightarrow \infty} \frac{(-2)^n + 3^n}{(-2)^{n+1} + 3^{n+1}} = \lim_{n \rightarrow \infty} \frac{3^n \left(\left(\frac{-2}{3}\right)^n + 1 \right)}{3^{n+1} \left(\left(\frac{-2}{3}\right)^{n+1} + 1 \right)}$$

$$= \lim_{n \rightarrow \infty} \frac{1}{3} \cdot \frac{\left(\frac{-2}{3}\right)^n + 1}{\left(\frac{-2}{3}\right)^{n+1} + 1} = \frac{1}{3} \cdot \frac{\lim_{n \rightarrow \infty} \left(\frac{-2}{3}\right)^n + 1}{\lim_{n \rightarrow \infty} \left(\frac{-2}{3}\right)^{n+1} + 1} = 1/3$$

6.26

$a \in \mathbb{R}$
$$\lim_{n \rightarrow \infty} a^{n^2 + n} \begin{cases} |a| > 1: +\infty \\ |a| = 1: 1 \\ a \in (-1, 1): \emptyset \end{cases}$$

$a > 1$, 1) $(a^{n^2 + n})$ je rostoucí, shora neomezená

2) je vybrana podpos. z (a^n)

$a = -1$
$$\lim_{n \rightarrow \infty} (-1)^{n^2 + n} = \lim_{n \rightarrow \infty} (-1)^{n(n+1)} = 1$$

$a < -1$:

$(a^{n^2 + n}) = \left(\left(\frac{n+1}{2}\right) \left(\frac{n+1}{2}\right) \right)$ je vybrana z (a^k) $\left(\frac{n(n+1)}{2}\right) = \left(\frac{n+1}{2}\right)$
 $\left(\frac{n+1}{2}\right)$ - 1. člen z $(a^k)^n$



$$\left(\sqrt{1 + \frac{1}{n}} + 2 - 3 \sqrt{1 + \frac{2}{n}} \right)$$

6.19

$$\lim_{n \rightarrow \infty} \sqrt{n} \left(\sqrt{n+1} + 2\sqrt{n} - 3\sqrt{n+2} \right)$$

(Note: Above the expression, there are handwritten annotations: $\sqrt{a-b} + \sqrt{a+b}$ and $\sqrt{a+b} - \sqrt{a-b}$ with arrows pointing to the terms.)

$$= \lim_{n \rightarrow \infty} \sqrt{n} \frac{(\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2})(\sqrt{n+1} + 2\sqrt{n} - 3\sqrt{n+2})}{\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}}$$

$$= \lim_{n \rightarrow \infty} \sqrt{n} (n+1 + 4n + 3\sqrt{n^2+n} - 9(n+2))$$

$$\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}$$

$$= \lim_{n \rightarrow \infty} \frac{\sqrt{n} (-17 - 4n + 4\sqrt{n^2+n})}{\sqrt{n+1} + 2\sqrt{n} + 3\sqrt{n+2}}$$

(Note: To the right of the fraction, there are handwritten annotations: $-4n + 4n\sqrt{1+\frac{1}{n}}$ and $4n(\sqrt{1+\frac{1}{n}} - 1)$ with arrows pointing to the terms in the numerator.)

$$= \lim_{n \rightarrow \infty} \frac{16n(n+1) - (6n^2 + 136n + 289)}{(\sqrt{n+1} + 2 + 3\sqrt{1+\frac{2}{n}})(4n+17+4\sqrt{n(n+1)})}$$

$$= \lim_{n \rightarrow \infty} \frac{-120n - 289}{(\sqrt{n+1} + 2 + 3\sqrt{1+\frac{2}{n}})(4n+17+4\sqrt{n(n+1)})}$$

$$= \lim_{n \rightarrow \infty} \frac{-120 - 289/n}{(\sqrt{n+1} + 2 + 3\sqrt{1+\frac{2}{n}})(4 + \frac{17}{n} + 4\sqrt{1+\frac{1}{n}})}$$

$$= \lim_{n \rightarrow \infty} \frac{-120 - 289/n}{(\sqrt{n+1} + 2 + 3\sqrt{1+\frac{2}{n}})(4 + \frac{17}{n} + 4\sqrt{1+\frac{1}{n}})}$$

$$= -120/4 = -30$$

(Note: A circled 'E' is written at the bottom right.)

6.14

$$\lim_{n \rightarrow \infty} \frac{1}{\sqrt{n}} \left(\sqrt{n+1} + \sqrt{2n} - \sqrt{3n+2} \right)$$

$$\lim_{n \rightarrow \infty} \left(\sqrt{1+\frac{1}{n}} + \sqrt{2} - \sqrt{3+\frac{2}{n}} \right)$$

$$= 1 + \sqrt{2} - \sqrt{3}$$

6.15

$$\lim_{n \rightarrow \infty} \sqrt[3]{n^3+n^2+1} - \sqrt[3]{n^3-n^2+1}$$

$$(a-b) \left(\underbrace{a^2+ab+b^2}_{\text{red}} \right) = a^3 - b^3$$

$$\Rightarrow \lim_{n \rightarrow \infty} \frac{2n^2}{\left(n^3+n^2+1 \right)^{2/3} + \left(n^3+n^2+1 \right)^{1/3} \left(n^3-n^2+1 \right)^{1/3} + \left(n^3-n^2+1 \right)^{2/3}}$$

$$\lim_{n \rightarrow \infty} \frac{2}{\left(1+\frac{1}{n}+\frac{1}{n^3} \right)^{2/3} + \left(1+\frac{1}{n}+\frac{1}{n^3} \right)^{1/3} \left(1-\frac{1}{n}+\frac{1}{n^3} \right)^{1/3} + \left(1-\frac{1}{n}+\frac{1}{n^3} \right)^{2/3}}$$

$$= \frac{2}{3}$$

$$6.13 \quad \lim_{n \rightarrow \infty} \sqrt{n^2 + (-1)^n} - \sqrt{n^2 + (-1)^{n+1}}$$

$$\lim \frac{\cancel{0} \cdot (-1)^n + (-1)^{n+1}}{\sqrt{n^2 + (-1)^n} + \sqrt{n^2 + (-1)^{n+1}}}$$

$$\lim 2(-1)^n / \left(\sqrt{n^2 + \frac{1}{n}} + \sqrt{n^2 + \frac{1}{n}} \right)$$

$$(a_{2n}) \quad \lim (a_{2n}) = 1$$

$$(a_{2n+1}) \quad \lim (a_{2n+1}) = -1 \Rightarrow \lim (a_n) \text{ neex!}$$

$$\lim \frac{(-1)^n}{n} = 0$$

$$0 = \lim_{n \rightarrow \infty} -\frac{1}{n} \leq \lim_{n \rightarrow \infty} \frac{(-1)^n}{n} \leq \lim_{n \rightarrow \infty} \frac{1}{n} = 0$$

$$\lim \frac{2n^2}{(n^3+n^2+1)^{2/3} + ((n^3+n^2+1)(n^3-n^2+1))^{1/3} + (n^3-n^2+1)^{2/3}}$$

$$\begin{aligned} | \quad (n^3+n^2+1)^{2/3} &= \left(n^3 \left(1 + \frac{1}{n} + \frac{1}{n^3} \right) \right)^{2/3} \\ &= n^2 \left(1 + \frac{1}{n} + \frac{1}{n^3} \right)^{2/3} \end{aligned}$$

$$\begin{aligned} \left[(n^3+n^2+1)(n^3-n^2+1) \right]^{1/3} &= \left[n^6 \left(1 + \frac{1}{n} + \frac{1}{n^3} \right) \left(1 - \frac{1}{n} + \frac{1}{n^3} \right) \right]^{1/3} \\ &= n^2 \cdot \left(1 + \frac{1}{n} + \frac{1}{n^3} \right)^{1/3} \left(1 - \frac{1}{n} + \frac{1}{n^3} \right)^{1/3} \end{aligned}$$