START 7:40

7.1 $\lim_{n\to\infty} \frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n}} \leq 1/2$

$\frac{1}{2} - \frac{1}{\sqrt{n}} = \frac{\sqrt{n}/2 - 1}{\sqrt{n}} \leq a_n = \frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n}} \leq \frac{\sqrt{n}/2}{\sqrt{n}} = 1/2$

$\lor$ $\frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n+1}}$

$\forall n \, \exists n' \geq n \quad a$

$\frac{\sqrt{n}/2}{\sqrt{n+1}} > \frac{\sqrt{n}/2}{\sqrt{n}}$

$\forall a > 0 : \; a - 1 \leq \lfloor a \rfloor$

$\frac{1}{2} = \lim_{n\to\infty} \frac{1}{2} - \underbrace{\boxed{\frac{100}{\sqrt{n}}}}_{\downarrow\, 0} \leq \lim_{n\to\infty} \underbrace{\frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n}}}_{= 1/2} \leq \lim_{n\to\infty} \frac{1}{2} = \frac{1}{2}$

1/100   2/100   99/100

$\sqrt{n}/2 \sim \mathbb{Z} + \frac{99}{100}$

?

$$\boxed{\frac{\sqrt{n}/2}{\sqrt{n+1}} \le \frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n}}} \quad \nexists\, n_0 : \forall n \ge n_0$$

$$\frac{\sqrt{n}/2}{\sqrt{n+1}} > \frac{\lfloor \sqrt{n}/2 \rfloor}{\sqrt{n}} \ge \frac{\sqrt{n}/2 - 1}{\sqrt{n}}$$

$$= \frac{\sqrt{n}/2 - 0.99}{\sqrt{n}}$$

$$n > \frac{n^2+n-(4n+4)}{\underbrace{\sqrt{n^2+n}+2\sqrt{n+1}}_{<n}} \sim \frac{n^2-3n}{2n}$$

$$n\left(\sqrt{n^2+n}+2\sqrt{n+1}\right) > n^2-3n+4$$

7.5

$$\lim_{n\to\infty} \sum_{k=n^2}^{(n+1)^2} \frac{1}{\sqrt{k}}$$

$$2 \le a_n = \sum_{k=n^2}^{(n+1)^2} \frac{1}{\sqrt{k}} \le \underbrace{\left((n+1)^2 - n^2 + 1\right)}_{2n+2} \cdot \frac{1}{n}$$

$\forall s_i \le \frac{1}{n}$, $i \in \{n^2, \dots, (n+1)^2\}$

$\ge \frac{1}{n+1}$

$\forall n$: $2 \le a_n \le 2 + 2/n$

$\lim 2 + 2/n = 2 \implies \lim a_n = 2$

7.8 a) $A = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^n = \frac{1}{\lim \left(\frac{n}{n-1}\right)^n} = 1/e$

b) $B = \lim_{n\to\infty} \left(1 - \frac{1}{n}\right)^{-n} = \frac{1}{A} = e$

$\left(1 + \frac{1}{n}\right)^n \to e$ $\qquad a_n = \boxed{\left(\frac{n-1}{n}\right)^n}$

$\boxed{\left(\frac{n+1}{n}\right)^n}$ $\qquad \lim_{n\to\infty} \left(\frac{n}{n-1}\right)^n = e$

$\lim_{n\to\infty} \left(\frac{n-1}{n}\right)^{n+1}$ $\qquad \lim_{n\to\infty} \underbrace{\left(1 + \frac{1}{n-1}\right)^{n-1}}_{e} \cdot \underbrace{\left(1 + \frac{1}{n-1}\right)}_{1} = e$

c) $\lim_{n\to\infty} \left(1 + \frac{(-1)^n}{n}\right)^{(-1)^n \cdot n} = e$

LICHÉ

$\lim \left(1 - \frac{1}{n}\right)^{-n} = B = e$

SUDÉ

$\lim \left(1 + \frac{1}{n}\right)^n = e$

VĚTA: $\mathbb{N} \setminus V = A_1 \cup A_2 \dots \cup A_k$ $\quad \forall i: \lim\limits_{k_i \in A_i} a_{k_i} = L$

$|V| < \infty$ $\quad \forall i: |A_i| = \infty$

$\Longrightarrow \lim a_n = L$

7.10 $\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^{3n+2} = e^{-6}$

$$\lim\left[\left(1-\frac{2}{n}\right)^{n}\right]\left[\left(1-\frac{2}{n}\right)^{n}\right]\left[\left(1-\frac{2}{n}\right)^{n}\right]\cdot\left(1-\frac{2}{n}\right)^{2}$$

$$\left(1-\frac{2}{n}\right)^{n} = \left(\left[1-\frac{1}{(n/2)}\right]^{n/2}\right)^{2} \Rightarrow e^{-2}$$

$$\lim_{n/2\to\infty}\left(1-\frac{1}{n/2}\right)^{n/2} = 7.8(a) = e^{-1}$$

$$\lim_{n\to\infty}\left(1-\frac{2}{n}\right)^{3n+2}$$

SUDÉ $\lim\left(1-\frac{1}{k}\right)^{6k+2} \sim e^{-6}$

LICHÉ $\lim\left[\left(1-\frac{1}{k+1/2}\right)^{k}\right]^{6} = e^{-6}$

e^{-1}

7.9.

$$\lim_{n\to\infty}\left(\frac{2n+5}{2n+3}\right)^{n+1} = e$$

$$\lim_{n\to\infty}\left(1+\frac{1}{n+3/2}\right)^{n+3/2-1/2}$$

$a \geq 1$

$\sqrt{a} \geq 1$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n+3/2}\right)^{n+1+\frac{1}{2}} \cdot \left(1-\frac{2}{2n+3}\right) \to e$$

$\to 1$

$\le$

$\downarrow e$

$$\lim \left(1+\frac{1}{n+2}\right)^{n+1} \le \left(1+\frac{1}{n+1}\right)^{n+2}$$

$$= \lim_{n\to\infty} \left(1+\frac{1}{n+2}\right)^{n+2} / \left(\frac{n+2}{n+3}\right)$$

$\to 1$

$m = n+2$

6.23

$$a^k - b^k = (a-b)\left(\sum_{i=1}^{k} a^i b^{n-i-1}\right)$$

$$a = \sqrt[k]{n^k+1}, \quad b = \sqrt[k]{n^k-1}$$

$$\lim_{n\to\infty} n^{k-1}\left(\sqrt[k]{n^k+1} - \sqrt[k]{n^k-1}\right)$$

$k \in \mathbb{N}$

$$\lim_{n\to\infty} \frac{\cdot 2}{\sum_{i=1}^{k}\left(\cancel{\frac{1}{n^k}(\ldots-1)}\ \underbrace{\frac{(n^k+1)}{n^k}\frac{(n^k+1)}{n^k}\cdots\frac{(n^k+1)}{n^k}}_{i \text{ tane}}\ \underbrace{\frac{(n^k-1)}{n^k}\cdots\frac{(n^k-1)}{n^k}}_{(k-i-1) \text{ tane}}\right)^{1/k}}$$

$\forall i$: elde $k-1$ çarpım

$$\lim \frac{2}{\sum_{i=1}^{k} \sqrt[k]{\left(1 \pm \frac{1}{n^2}\right)^{k-1}}} = 2/k$$

i tane $(+)$, $(n-1-i)$ tane $(-)$

$$\lim \sqrt[k]{\underbrace{\left(1 \pm \frac{1}{n^2}\right)}_{\downarrow\, 1} \cdots\cdots \underbrace{\left(1 \pm \frac{1}{n^k}\right)}_{\downarrow\, 1}} = 1$$