START 7:40

f. 1 $\lim_{n\to\infty} a_n$ $a_1 = 10$, $a_{n+1} = a_n - 1/\sqrt{n}$

$$a_n = 10 - n/\sqrt{n}$$

$$a_2 = 10 - 1/\sqrt{2} \qquad a_3 = 10 - 1/\sqrt{2} - 1/\sqrt{3}$$

$$a_n = 11 - \boxed{\sum_{k=1}^{n} 1/\sqrt{k}} \qquad \boxed{\lim_{n\to\infty} a_n = -\infty}$$

$$\lim_{n\to\infty} \sum_{k=1}^{n} 1/\sqrt{k} = \infty \ ?$$

$$+\infty \geq \lim \sum_{k=1}^{n} 1/\sqrt{k} \geq \lim_{n\to\infty} \sqrt{n} = +\infty$$

$$b_n := \sum_{k=1}^{n} 1/\sqrt{k} \geq \sum_{k=1}^{n} 1/\sqrt{n} = n \cdot 1/\sqrt{n} = \sqrt{n}$$

$$\geq \sum_{k=1}^{\infty} 1/k = +\infty$$

8.2 (ii) $\lim_{n\to\infty} a_n$ $a_1 = 5$

$\forall n \in \mathbb{N}$ $a_n = 5$ $a_2 = 2\cdot 5 - 5 = 5$

$$\boxed{a_{n+1} = 2\cdot a_n - 5}$$

8.2 (i) $\boxed{a_1 = 4}$

$\lim a_n = -\infty$

< 5

$a_{n+1} < a_n$ $\forall n \in \mathbb{N}$

Dk: indukcí $n=1$

$3 = a_2 < a_1 = 4$

$n = n_0 \geq 2$ $a_n < a_{n-1}$ $\forall n \leq n_0$

$a_{n_0+1} < \boxed{a_{n_0}} < a_{n_0-1} < a_1 = 5$

$a_{n_0+1} = 2\cdot a_{n_0} - 5 < a_{n_0}$

8.2 (iii) $a_1 = 6$ > 5

$a_{n+1} > a_n$ $\forall n \in \mathbb{N}$

$7 = a_2 > a_1 = 6$

$n = n_0 \geq 2$ $\geq IP$ $a_{n_0} > 6$

$a_{n_0+1} = 2a_{n_0} - 5 > a_{n_0} + 1$

$\Rightarrow$ $\lim_{n\to\infty} a_n = +\infty$

Krok 1) $\boxed{a_n \leq 4}$ $\forall n \in \mathbb{N}$

dk: Indukcí dle n, $n=1$ $a_1 = 4$ ✓

$n = n_0+1$ $a_{n_0+1} = 2a_{n_0} - 5 \leq 8 - 5 = 3 \leq 4$

Krok 2) : $a_{n+1} < a_n$

$$a_{n+1} = 2\boxed{a_n} - 5 \leq a_n + \underbrace{(a_n - 5)}_{\leq -1} \leq a_n - 1$$

$a_1 = 4, \quad a_{n+1} = 2 \cdot a_n - 5$

Tvrzení: $a_{n+1} < a_n$ $\boxed{n=1 \checkmark}$ IP $a_{n'+1} < a_{n'} \quad \forall n' \leq n$

Chci $\boxed{a_{n+2} < a_{n+1}}$

$a_{n+2} = \boxed{2a_{n+1} - 5 < a_{n+1}}$

$\Updownarrow$

$a_{n+1} < 5$

$$a_{n+1} \underset{\text{IP}}{<} a_n \underset{\text{IP}}{<} a_{n-1} \underset{\text{IP}}{<} a_{n-2} \cdots \underset{\text{IP}}{<} a_1 = 4 < 5 \checkmark$$

§ 5 (i) "PODÍLOVÉ KRITERIUM" (a_n) nenulových čísel

$$L = \lim \left|\frac{a_{n+1}}{a_n}\right| < 1 \implies \lim_{n\to+\infty} a_n = 0$$

$\in [0,1)$

$$\Updownarrow$$

$$\lim_{n\to\infty} |a_n| = 0$$

z def limity:
$\forall \varepsilon > 0 \ \exists n_0 \ \forall n \geq n_0$

$\varepsilon_H := \frac{1-L}{2}$

$\varepsilon_D := \frac{L-1}{2}$

$$L - \varepsilon_D \leq \boxed{\left|\frac{a_{n+1}}{a_n}\right| \leq L + \varepsilon_H} = \frac{1}{2} + \frac{L}{2}$$

$$|a_n| \geq |a_{n_0}| \left(\frac{L+1}{2}\right)^{n-n_0}$$

$$|a_{n+1}| < \left(\frac{1+L}{2}\right)|a_n|$$

$a_n = (-1)^n (1/2)^n$

$a_1 = -1 \cdot 1/2$
$a_2 = +1 \cdot (1/2)^2$
$a_3 = -1 \cdot (1/2)^3$
$a_4 = -1 \cdot (1/2)^4$
$a_5 = +1 \cdot (1/2)^5$
$a_6 = +1 \cdot (1/2)^6$

$\Vert$

0

TVRZENÍ: $\exists n_0 : \forall n \geq n_0 : \ |a_n| \leq |a_{n_0}| \left(\frac{1+L}{2}\right)^{n-n_0}$

Dk: Indukcí dle n; $n = n_0$ $\quad n = n_0 : |a_{n_0}| \leq |a_{n_0}|$

$n+1 > n_0$:

$$|a_{n+1}| \leq |a_n| \cdot \left(\frac{1+L}{2}\right) \leq |a_{n_0}| \cdot \left(\frac{1+L}{2}\right)^{n-n_0+1}$$

IP: $|a_n| \leq |a_{n_0}| \cdot \left(\frac{1+L}{2}\right)^{n-n_0}$. $\quad$ CHCI: $|a_{n+1}| \leq |a_{n_0}| \left(\frac{1+L}{2}\right)^{n+1-n_0}$

$\exists n_0 : \forall n \geq n_0$

$$0 \leq |a_n| \leq |a_{n_0}| \left(\frac{1+L}{2}\right)^{n-n_0}$$

$\downarrow \lim \qquad \downarrow \qquad \downarrow \lim$

$$0 \qquad \cancel{0} \qquad |a_{n_0}| \cdot 0 = 0$$

BTW
$$\lim \frac{\log(n+1)}{\log(n)} = 1$$
$a_n = \log(n)$

$a_n = 1/n$
$\lim(a_n) = 0$

$$\frac{a_{n+1}}{a_n} = \frac{n}{n+1} \longrightarrow 1$$

$\boxed{a_n = 3}$ $\quad \lim \frac{a_{n+1}}{a_n} = 1$
$\lim(a_n) = 3$

$\S.5$ (ii) : $\underbrace{\lim \left|\frac{a_{n+1}}{a_n}\right| > 1} \rightsquigarrow \lim |a_n| = +\infty$

$\S.10$: $\lim_{n\to\infty} \frac{n^n}{2^n \cdot n!}$

$a_n = |a_n|$

$a_n = |a_n|$

$$\lim_{n\to\infty} \frac{a_{n+1}}{a_n} = \frac{(n+1)^{n+1} \cdot \cancel{2^n} \cdot \cancel{n!}}{n^n \cdot \cancel{2^{n+1}} \cancel{(n+1)!}} = \frac{1}{2} \lim \left(\frac{n+1}{n}\right)^n = e/2 > 1$$

$\Rightarrow$ P.K vime, 2i $\lim |a_n| = +\infty$

$\lim (a_n)$ $\boxed{\nexists}$

$\S.7$

$\lim (b_n) = \lim \frac{a^n}{n!}$ $\quad a \in \mathbb{R}$

$$\lim_{n\to\infty} \left|\frac{b_{n+1}}{b_n}\right| = \lim_{n\to\infty} \left|\frac{a}{(n+1)}\right| = |a| \cdot \lim_{n\to\infty} \left(\frac{1}{n+1}\right) = 0$$

P.K. $\rightsquigarrow \lim (b_n) = 0$