9.10

$$\lim_{x\to\infty} \frac{\sqrt{x+\sqrt{x+\sqrt{x}}}}{\sqrt{x+1}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{x+\sqrt{x+\sqrt{x}}}{x+1}} = \lim_{x\to\infty} \sqrt{\frac{1+\sqrt{1/x+\frac{\sqrt{x}}{x^2}}}{1+1/x}}$$

$$= \lim_{x\to\infty} \sqrt{\frac{1+\sqrt{1/x+\sqrt{1/x^3}}}{1+1/x}} = \sqrt{\lim_{x\to\infty} \frac{1+\sqrt{1/x+\sqrt{1/x^3}}}{1+1/x}}$$

$\lim_{x\to a} f(g(x)) \overset{?}{=} c$ VOLSF(A) $\sqrt{1} = 1$

$\lim_{x\to a} g(x) = b$ (B) $g(x) \neq b$ jen pro $x = a$ na nějakém H_a

$\lim_{y\to b} \boxed{f(y)} = c$ (A) f je spojitá v bodě b (AKA $f(b) = c$)

9.26

$$\lim_{x\to 0} \frac{1-\cos x}{x^2}$$

$$\lim_{x\to 0} \frac{\sin x}{x} = 1$$

$$\lim_{x\to 0} \frac{e^x - 1}{x} = 1$$

"e^x"

$$\lim_{x\to 0} \frac{\cancel{1} - \cos^2(x/2) + \sin^2(x/2)}{x^2}$$

$$\lim_{x\to 0} \frac{\cos x}{1} = 1$$

$(\sin x)' = \cos x$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$1 = \cos^2\alpha + \sin^2\alpha$

$$\lim_{x\to 0} \frac{\cancel{\cos^2(x/2)} + 2\sin^2(x/2)}{x^2}$$

VOLSF (x^2 je spojitá v 1)

$$\frac{1}{2}\lim_{x\to 0} \frac{\sin^2(x/2)}{(x/2)^2} = \left(\lim_{x\to 0} \frac{\sin(x/2)}{x/2}\right)^2$$

$$\lim_{x\to 0} \frac{\sin(x/2)}{x/2} = 1$$

VOLSF

$f = \frac{\sin y}{y}$ $\lim_{y\to 0} f(y) = 1$

$g = x/2$ $\lim_{x\to 0} x/2 = 0$

(B) g je prostá f

$f = y^2$ $\lim_{y\to 1} y^2$

$g = \frac{\sin(x/2)}{x/2}$ $\lim_{x\to 0} g(x) = 1$

$$\lim_{x\to 0} \frac{(1-\cos x)(1+\cos x)}{x^2\,(1+\cos x)}$$

$$\boxed{\lim_{x\to 0} \frac{\overset{\sin^2 x}{\overset{\|}{1-\cancel{\cos}^2 x}}}{x^2}} \cdot \frac{1}{1+\cos x}$$

$$\|$$

$$1 \cdot \underbrace{\lim_{x\to 0} \frac{1}{1+\cos x}}_{1/2} = 1/2$$

9.11

$$\lim_{x\to 4} \frac{\boxed{\sqrt{1+2x} - 3}}{\boxed{\sqrt{x} - 2}} = \lim_{x\to 4} \frac{(\sqrt{1+2x} - 3)(\sqrt{x}+2)}{x-4}$$

$$= \lim_{x\to 4} \frac{\sqrt{2x^2+x} - 3\sqrt{x} + 2\sqrt{1+2x} - 6}{x-4}$$

$$\lim_{x\to 4} \frac{(\sqrt{1+2x} - 3)}{(\sqrt{x} - 2)} \frac{(\sqrt{1+2x}+3)}{(\sqrt{1+2x}+3)}$$

$$\|$$

$$\lim_{x\to 4} \frac{2(x-4)}{(\sqrt{x} - 2)(\sqrt{1+2x}+3)} = \lim_{x\to 4} \frac{2(\sqrt{x}+2)\cancel{(\sqrt{x}-2)}}{\cancel{(\sqrt{x}-2)}(\sqrt{1+2x}+3)}$$

$$= \lim_{x\to 4} \frac{2(\sqrt{x}+2)}{(\sqrt{1+2x}+3)} = 8/6 = 4/3$$

9.23 b

$\alpha, \beta \in \mathbb{R}$

$\beta \neq 0$

$$\lim_{x \to 0} \frac{\sin(\alpha x)}{\sin(\beta x)}$$

$$\lim_{x \to 0} \frac{\frac{\sin(\alpha x)}{\alpha x} \cdot \beta}{\frac{\sin(\beta x)}{\beta x} \cdot \alpha} = \frac{\alpha}{\beta} \lim_{x \to 0} \frac{\overbrace{\left[\frac{\sin(\alpha x)}{\alpha x}\right]}^{1}}{\underbrace{\left[\frac{\sin(\beta x)}{\beta x}\right]}_{1}}$$

$= \alpha / \beta$

9.2 g

$$\lim_{x \to 0} \frac{\ln(x^2 - 5x + 1)}{x^2 - 5x} \cdot (x - 5)$$

$$\lim_{x \to 0} \frac{\ln(1+x)}{x}$$

$0 \notin D_f$

$= -5$

VOLSF $f = \ln \frac{1+y}{y}$ $\lim_{y \to 0} f(y) = 1$

(B) $g = x^2 - 5x$. $\lim_{x \to 0} g(x) = 0$

$g(x) \neq 0$ pro nějaké $\varepsilon > 0$

$\forall x \in (-\varepsilon, \varepsilon) \setminus \{0\}$

?

$x^2 - 5x = 0$ $x \begin{cases} 0 \\ 5 \end{cases}$

např. $\varepsilon = 5$ $\Rightarrow$ na $A := (-5, 5) \setminus \{0\}$

$g(x) \neq 0$