Start 7:42

11.14 $f(x) = [\log_a(x^2)]^3 \quad a > 0$

VDSF: $y^3 \quad y = \log_a x^2 \quad \boxed{x \neq 0}$

$f'(x) = 3[\log_a(x^2)]^2 \cdot [\log_a(x^2)]'$

VDSF
$f(y) = \log_a y$
$g(x) = x^2$
f je dif v $y = g(x)$
pro $\forall x \neq 0$

$(\log_a y)' = 1/y \cdot \ln(a)$

$$\boxed{\frac{6}{\ln(a)} \cdot \frac{[\log_a(x^2)]^2}{x}}$$

$$= \frac{6}{x \cdot \ln(a)} \left[\frac{\ln(x^2)}{\ln(a)}\right]^2 = \frac{6[\ln(x^2)]^2}{x \cdot [\ln(a)]^3} = \frac{24[\ln(x)]^2}{x \cdot [\ln(a)]^3}$$

11.13: $f(x) = |x+2| e^{-\frac{1}{x}} \quad x \neq 0$

$x \in (-\infty, -2)$ A

$x = -2$ C $\quad f(-2) = 0$

$x \in (-2, +\infty) \setminus \{0\}$ B

A $f'(x) = [(-x-2)e^{-1/x}]' = (-x-2) \cdot (e^{-1/x})' - e^{-1/x}$

$g = e^y \quad f = -\frac{1}{x} \quad \rightsquigarrow (e^{-1/x})' = e^{-1/x} \cdot \left(-\frac{1}{x}\right)' = e^{-1/x} \cdot \left(\frac{(-1)(-1)}{x^2}\right)$

$$= e^{-1/x}\left[\frac{-x-2}{x^2} - 1\right]$$

(B)

$$f'(x) = \left[(+x+2)e^{-1/x}\right]' = (+x+2)\cdot\left(e^{-1/x}\right)' + e^{-1/x}$$

$g = e^y \quad f = -\frac{1}{x} \quad \rightsquigarrow \left(e^{-1/x}\right)' = e^{-1/x}\cdot\left(-\frac{1}{x}\right)' = e^{-1/x}\cdot\left(\frac{(-1)(-1)}{x^2}\right)$

$$= e^{-1/x}\left[\frac{+x+2}{x^2} + 1\right]$$

(C) $x = -2$

$$f'(-2) = \lim_{x\to -2} \frac{f(x)}{x-(-2)}$$

$$\lim_{x\to -2+} \frac{x+2}{x+2}\cdot e^{-1/x}$$

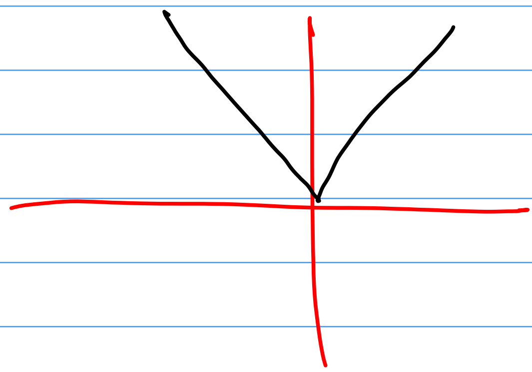
$\lim_{x\to -2} e^{-1/x} = e^{1/2}$

$$\lim_{x\to -2-} \frac{-x-2}{x+2}\cdot e^{-1/x}$$

$= -e^{1/2}$

$\Rightarrow f'(-2)$ neex., ex. derivace zprava/zleva

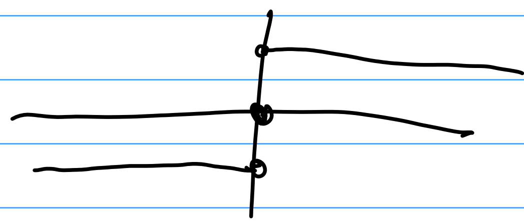
$f(x) = |x|$



$f(0) = 0$

$f'(0) =$ neex.

$f(x) = \mathrm{sign}(x)$



$f(0) = 0$

$f'(0) = \infty$

11.12: $f(x) = \ln[\ln(\sin x)]$ $\quad D_f = \emptyset$

$\sin x \leq 1 \Rightarrow \ln(\sin x) \leq 0$

$\ln(y)$ $\quad y := \ln(\sin x)$ je nedef. $\forall x \in \mathbb{R}$

11.11 $f(x) = \ln(\ln(\ln(x)))$ $\quad D_f = (e, \infty)$

$x > 0$

$x > 1$

$x > e$

$$f'(x) \overset{VZSF}{=} \frac{1}{\ln(\ln x)} \cdot (\ln \ln x)' \overset{VZSF}{=} \frac{1}{\ln(\ln x)} \cdot \frac{1}{\ln x} \cdot (\ln(x))' = \frac{1}{\ln(\ln x) \cdot \ln(x) \cdot x}$$

$$\left(f[g(h(x))]\right)'(a) \overset{VDSF(o,i)}{=}$$

$o(y)$ diferencovatelná v bodě $g(h(a))$

$i(x)$ dif. v bodě a

$o :=$ vnější

$i :=$ vnitřní

$i := g(h(x))$

$o = f(y)$

$$= f'(g(h(a))) \cdot (g(h(x)))'(a) \overset{VDSF(o_2, i_2)}{=}$$

$o_2 := g(y)$

$i_2 := h(x)$

$$f'(g(h(a))) \cdot g'(h(a)) \cdot h'(a)$$

$\left[\begin{array}{lll} f \text{ dif. v} & g(h(a)) \\ g \text{ dif. v} & h(a) \\ h \text{ dif. v} & a \end{array}\right], a \in D_{(f \circ g \circ h)'}$

o_2 byla dif. v $h(a)$

i_2 byla dif. v a

Která z vět o přírůstku funkce je NEJ?

(kap. 4.6)

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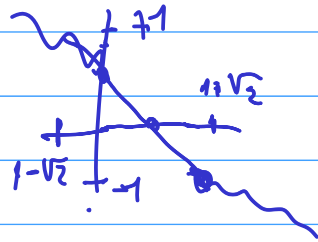
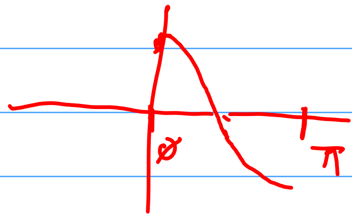
11.19. $f(x) = \arccos\left(\frac{1-x}{\sqrt{2}}\right)$

$D_f = [1-\sqrt{2}, 1+\sqrt{2}]$

$(\arccos)'(y) = \frac{-1}{\sqrt{1-y^2}}$ $\quad y \in (-1,1)$



$D_{\arccos(y)} = [-1,1]$



$$f'(x) \overset{VDSF}{=} \frac{-1}{\sqrt{1-\frac{(1-x)^2}{2}}} \cdot \left(-\frac{1}{\sqrt{2}}\right)$$

$$= 1/\sqrt{2-(1-x)^2}$$

$$= 1/\sqrt{1+2x-x^2}$$