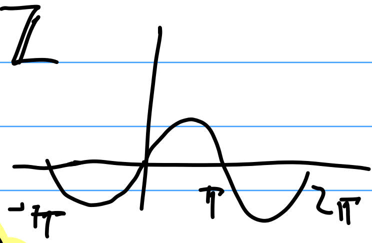
12.9 (ii) $f(x) = x/\sin x$

pro která $x \in \mathbb{R}$ je f spojitá
tam kde nespojitá druh nespojitosti jde?

$\sin x = 0 \iff x = \underbrace{k\pi}$ $k \in \mathbb{Z}$

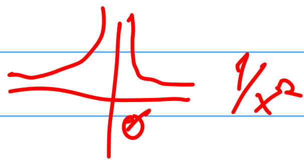
f je spojitá na $\mathbb{R} \setminus \{k\pi : k \in \mathbb{Z}\}$



$$= \bigcup_{k \in \mathbb{Z}} (k\pi, (k+1)\pi)$$

1) odstranitelná
2) skok
3) ostatní...

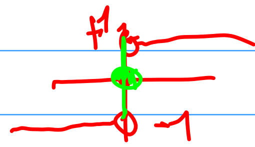
$\exists \lim_{x \to a} f(x) \iff$ $\underline{a \in \mathbb{R}}$ $\exists \lim_{a+} f$ & $\exists \lim_{a-} f$ & $L_- = L_+$



• bod $\underline{a \in \mathbb{R}}$ je odstranitelná nespojitost pro $f(x)$
pokud $\exists \lim_{x \to a} f(x) \in \mathbb{R}$

$$\lim_{x \to 0 \cdot \pi} f(x) = \lim_{x \to 0} \frac{x}{\sin x} = 1$$

sign(x)

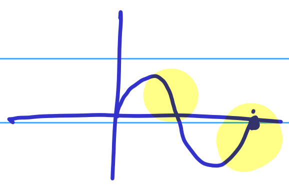


$\forall z \in \mathbb{Z} \setminus \{0\}$: nespojitost $f(x)$ v $z \cdot \pi$ není ani skok, ani odstranitelná

• bod $a \in \mathbb{R}$ je skok $f(x)$, pokud $\exists \lim_{x \to a+} f(x) =: L^+ \in \mathbb{R}$

$\lim_{x \to a-} f(x) =: L^- \in \mathbb{R}$

$L^- \neq L^+$



$z \in \mathbb{Z}^+$ [illegible]

$$\lim_{x \to z \cdot \pi -} \frac{x}{\sin x} = \begin{cases} z \text{ sudé: } & -\infty \\ z \text{ liché: } & +\infty \end{cases}$$

$z \in \mathbb{Z}^-$

$\lim_{x \to z \cdot \pi +}$

z liché, z sudé $= (x) \to$ mil

$+\pi \cdot s \leftarrow x$

$= (x) \to$ mil

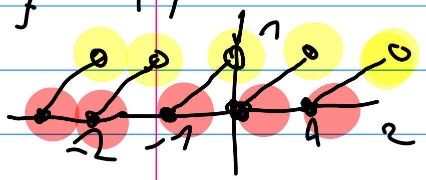
$-\pi \cdot s \leftarrow x$

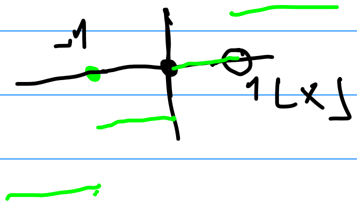
12.9 (iii) $f(x) := x - \lfloor x \rfloor$ $D_f \in \mathbb{R}$ $\frac{(1-3x)}{x^{2/3}(1-x)^{1/3}}$

jaké má body nespojitosti? $\mathbb{Z}$

$\lfloor x \rfloor := \max_{a \in \mathbb{Z}} a : a \leq x$ Př. $\lfloor -0.5 \rfloor = -1$

$H_f = [0, 1)$





$\forall a \in \mathbb{Z}$: nespojitost $f(x)$ v a je SKOK

$\lim_{x \to a-} f(x) = +1 \neq 0 = f(a)$ mil $+\infty <- x$

Zpět v F:35

který typ nespojitosti je nej?

12.12: vyšetřete lokální extrémy

$$f(x) = x^{\frac{1}{3}} (1-x)^{2/3}$$

$$f'(x) = \frac{1}{3} x^{-2/3} (1-x)^{2/3}$$

$$- x^{\frac{1}{3}} \cdot \frac{2}{3} (1-x)^{-1/3}$$

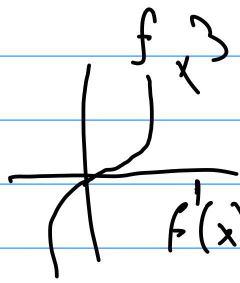
$$= \frac{1}{3} \cdot \left(\left[\frac{1-x}{x}\right]^{2/3} - 2 \cdot \left(\frac{x}{1-x}\right)^{1/3} \right)$$

$a = b$ x^3 prostá

$\Updownarrow$

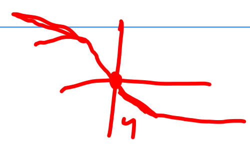
$a^3 = b^3$

$$\left(\frac{1-x}{x}\right)^{2/3} = 2 \cdot \left(\frac{x}{1-x}\right)^{1/3}$$



$f'(x) = 3x^2$

$x = 0$



$x \neq 0$
$x \neq 1$

$$\left(\frac{1-x}{x}\right)^2 = 8\,\frac{x}{1-x}$$

$$\left(\frac{1-x}{x}\right)^3 = 8$$

$$\frac{1-x}{x} = 2 \iff 1 = 3x \iff x = 1/3$$

$$\frac{1}{3}\cdot\lim_{x\to 0+} = +\infty$$

$$\lim_{x\to 0-} = +\infty$$

$$\frac{1}{3}\lim_{x\to 1+} = +\infty$$

$$\frac{1}{3}\lim_{x\to 1-} = -\infty$$

Darbouxova věta: $f: D_f \to \mathbb{R}$, $b \in D_f$

1) f je spojitá v b zprava/zleva
2) $\exists \varepsilon > 0$: f diferencovatelná na $(b, b+\varepsilon)$ / $(b-\varepsilon, b)$
3) $\exists \lim\limits_{x\to b+} f'(x) =: L$

$$f'_{+}(b) = L$$

$$f'(0) = +\infty$$

$\rightsquigarrow$ body podezřelé z extrému: $x = 1/3$, $x = 1$

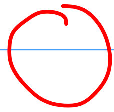
f': $+ \quad + \quad - \quad +$ (body $1/3$, 1)

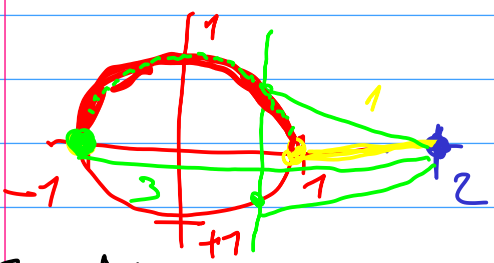
$$\frac{1}{3}\cdot\frac{(1-3x)}{x^{2/3}(1-x)^{1/3}}$$

$x = 1/3$ ostré lok. max

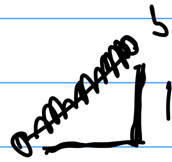
$x = 1$ ostré lok. min

12.2 } Najděte nejdelší a nejkratší vzdálenost $(2,0)$ od $x^2+y^2=1$





Euklidovská vzdálenost $a,b\in\mathbb{R}^2$



$$\sqrt{(a_x-b_x)^2+(a_y-b_y)^2}$$

$x^2\le 1 \iff |x|\le 1$

$\Uparrow$

$x^2+y^2=1 \Rightarrow y^2=1-x^2$

extrémy $\sqrt{(x-2)^2+y^2}$

$||$

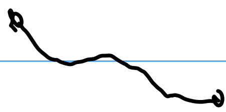
$D_f := \langle -1,1\rangle$

$$f(x)=\sqrt{(x-2)^2+1-x^2}=\sqrt{5-4x}$$

$$f'(x) = -2\cdot 1/\sqrt{5-4x}$$

$f'(x)<0$ $\quad$ f je klesající

f nemá lokální extrémy



takže $1=f(1)$ je $\underline{MIN}$ $\quad -1 \quad +1$

$3=f(-1)$ je $\underline{MAX}$

$$x^2+y^2=1 \Longrightarrow x^2\le 1$$

$\Updownarrow$

$|x|\le 1$

$(x,y)\in\mathbb{R}$

pouze když $x\in\langle -1,1\rangle$

$\forall x\in\langle -1,1\rangle$

$y:=\sqrt{1-x^2}$

$x^2+y^2=1$