Game Theory

Spring term 2022/23, FJFI ČVUT

2^{nd} homework assignment

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1) Two players are sitting at a completely empty rectangular table that has its side-lengths a and b centimeters, respectively. Each of the players has a supply of 1 CZK coins that is more than enough to completely cover the table; Note that the radius of a 1 CZK coin is 1 centimeter. Now consider the following impartial normal game (i.e., the two players alternate and the one that cannot move loses):

   - in each turn, the corresponding player places one new coin on the table,
   - when placing a new coin, it must be placed fully on the table,
   - no coin that has been placed before shall be moved, and
   - the coins on the table may touch but cannot be stacked over each other.

   Based on the parameters a and b, determine when does the first player has a winning strategy.

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2) Recall the game S-Pick-Up-Brick with n bricks, where S is a finite set containing positive integers: it is an impartial normal game that starts with n bricks, and in each turn the current player removes a bricks for some $a \in S$.

a) For $S_k := \{1, 2, \dots, k\}$, let α_n^k be the S_k-Pick-Up-Brick game with n bricks. For every $k \in \mathbb{N}$ and $n \in \mathbb{N}$, determine $a_n^k \in \mathbb{N}_0$ such that $\alpha_n^k \equiv \star a_n^k$.

b) Let β_n be the $\{1, 4, 5\}$-Pick-Up-Brick game with n bricks. Based on the parameter n, determine when does the first player has a winning strategy.

c) For every $n \in \mathbb{N}$, determine $b_n \in \mathbb{N}_0$ such that $\beta_n \equiv \star b_n$.

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3) Recall the game Chop: it is an impartial normal game that starts with $n \times m$ plank that is secured only at the lower left corner. On each turn, a player must either make a vertical or horizontal chop of the plank, and then the piece no longer connected to the lower left corner falls off into the water. We denote the Chop game with $n \times m$ plank by $\gamma_{m,n}$.

   For every $m \in \mathbb{N}$ and $n \in \mathbb{N}$, prove that $\gamma_{m,n} \equiv \star(m-1) + \star(n-1)$.