$D_f = D_g$

$f: \mathbb{N} \to \mathbb{R}$ (D_f) $\qquad g: \mathbb{N} \to \mathbb{R}$ (D_g)

$f = O(g)$ pokud $\exists c_1 > 0$ t. ž. $f(n) \leq c_1 \cdot g(n)$

$f = \Omega(g)$ $\quad \exists c_2 > 0 \quad f(n) \geq c_2 \cdot g(n)$

$f = \Theta(g)$ pokud $f = O(g)$ a $f = \Omega(g)$

alternativně: $f = \Omega(g) \iff g = O(f)$

$f = o(g)$ pokud $\lim_{n \to \infty} \frac{f(n)}{g(n)} = 0$ $\qquad f = \omega(g)$ pokud $g = o(f)$

SLOŽITOST ALGORITMŮ $\left\langle \begin{array}{l} \text{ČASOVÁ} \\ \text{PAMĚŤOVÁ} \end{array} \right.$

Příklad: spočítat $n!$

ČAS: $\Theta(n)$

PAMĚŤ: $O(1)$

- $r \leftarrow 1$
- for i 1 to n: $r \leftarrow \boxed{r * i}$
- return r

RAM

random access machine

třídění n čísel $a_1 \dots a_n$

for $i := 1$ to n

$m := \min(a_i, \dots a_n)$, $j :=$ index toho m

prohoď (a_i, m)

$c(n + (n-1) + (n-2) \dots + 1) = O(n^2)$

Merge Sort $\leadsto O(n \log n)$

$T(n)$ Merge Sort$(a_1 \ldots a_n)$: if $n=1$ return a_1 else:

$2 \cdot T(\frac{n}{2})$ $\{$ $x_1 \ldots x_{\frac{n}{2}} :=$ MergeSort$(a_1 \ldots a_{\frac{n}{2}})$

$y_1 \ldots y_{\frac{n}{2}} :=$ MergeSort$(a_{\frac{n}{2}+1} \ldots a_n)$ $\quad n = 2^i$

$O(n) \longleftarrow$ $b_1 \ldots b_n :=$ Merge$(\boxed{x_1 \ldots x_{\frac{n}{2}}}, \boxed{y_1 \ldots y_{\frac{n}{2}}})$

return $b_1 \ldots b_n$

Merge(L, P): kde L setříděné $\leadsto$ setříděné $L \cup P$

$i, j := 1$

for $c := 1$ to len(L) + len(P)

$O(\text{len}(L) + \text{len}(P))$

if $L_i < P_j$:

$r_c := L_i$

$i += 1$

if $i >$ len(L): break

else:

$r_c := P_j$

$j += 1$

if $j >$ len(P): break

$i = \text{len}(L)$

$j = \text{Len}(P)$

$L_4 \ldots L_3$ $\emptyset$

return $\boxed{r_1 \ldots r_c}, L_{i+1} \ldots L_{\text{len}(L)}, P_j \ldots P_{\text{len}(P)}$

$P = 134$

$L\ 256$

$$T(n) = 2 \cdot T\left(\frac{n}{2}\right) + n$$

$$= 2\left(2T\left(\frac{n}{4}\right) + \frac{n}{2}\right) + n$$

$$= 4 \cdot T\left(\frac{n}{4}\right) + 2n$$

$$= 8 \cdot T\left(\frac{n}{8}\right) + 3n = \ldots = 2^i T\left(\frac{n}{2^i}\right) + in$$

$i_s := \log_2 n$ wocich

$$\ldots \overset{\downarrow}{=} 2^{i_s} T(1) + i_s \cdot n =$$

$$n + \log n \cdot n \Rightarrow T(n) = \Theta(n \log n)$$

$a \times b$ násobení čísel (dlouhých)

$a_1 \dots a_k$
$d_1 \dots d_k$
$O(k)$

$$
\begin{array}{r}
a_1 \dots \dots a_n \\
\times\ b_1 \dots \dots b_n \\
\hline
b_1 \times \ \boxed{a_1 \dots \dots a_n} + \\
b_2 \times \ \boxed{\qquad\qquad\quad} \ 0 + \\
\vdots
\end{array}
\rightsquigarrow \Theta(n^2)
$$

$\Theta(n^2)$ násobení
$\Theta(n^2)$ sčítání

$$X = \underbrace{x_1 x_2 \dots x_{\frac{n}{2}}}_{A} \underbrace{\dots x_n}_{B} \qquad Y = \underbrace{y_1 y_2 \dots y_{\frac{n}{2}}}_{C} \underbrace{\dots y_n}_{D}$$

$$X = 10^{n/2} \cdot A + B \qquad Y = 10^{n/2} \cdot C + D$$

$$XY = (10^{n/2} A + B)(10^{n/2} C + D)$$
$$= 10^n \boxed{AC} + 10^{n/2}(\boxed{BC} + \boxed{AD}) + \boxed{BD}$$

$$T(1) = 1$$

$$T(n) = 4\,T\left(\frac{n}{2}\right) + z \cdot n$$
$$= 4\left(4T\left(\frac{n}{4}\right) + z\left(\frac{n}{2}\right)\right) + z \cdot n = 16\,T\left(\frac{n}{4}\right) + 3zn \qquad (2^i - 1)$$
$$64\,T\left(\frac{n}{8}\right) + 16 z \frac{n}{4} + 3zn = 64\,T\left(\frac{n}{8}\right) + 7z \cdot n$$

$$= \dots = \underbrace{4^i T\left(\frac{n}{2^i}\right)}_{2^{2\log_2 n} = n^2} + \underbrace{(2^i - 1)}_{n} \Theta(n)$$

$i = \log_2 n$

$$n \cdot \Theta(n) = \Theta(n^2)$$

AC, BD

$$z = (A+B)(C+D) = AC + \underline{BC + AD} + BD$$

$$z - AC - BD = (BC + AD)$$

$$xy = (10^{n/2} A + B)(10^{n/2} C + D)$$

$$= 10^n \boxed{AC} + 10^{n/2} \boxed{BC + AD} + \boxed{BD}$$

$BC + AD$

$$T(n) = 3 \cdot T\left(\frac{n}{2}\right) + q \cdot n$$

$$= \dots = \dots$$

$$\Theta(n^{\log_2 3}) = O(n^{1.6})$$

$a = 3,\ b = 2,\ c = 1$

MASTER THEOREM:

Rekurence $T(1) = 1$, $T(n) = a \cdot T(n/b) + \Theta(n^c)$

podmínky $a \geq 1$, $b > 1$, $c \geq 0$ má řešení

- $a = b^c \rightsquigarrow T(n) = \Theta(n^c \log n)$
- $a < b^c \rightsquigarrow T(n) = \Theta(n^c)$
- $a > b^c \rightsquigarrow T(n) = \Theta(n^{\log_b a})$