

$$D_f = D_g$$

$$f: \mathbb{N} \rightarrow \mathbb{R}$$

$$g: \mathbb{N} \rightarrow \mathbb{R}$$

$$f = O(g)$$

$$f = \Omega(g)$$

$$f = \Theta(g)$$

pokud $\exists c_1 > 0$ t. \exists $f(n) \leq c_1 g(n)$

$$\exists c_2 > 0$$

$$f(n) \geq c_2 g(n)$$

pokud $f = O(g)$ a $f = \Omega(g)$

alternativně, $f = \Omega(g) \Leftrightarrow g = O(f)$

$$f = o(g) \text{ pokud } \lim_{n \rightarrow \infty} \frac{f(n)}{g(n)} = 0$$

$$f = \omega(g) \text{ pokud } g = o(f)$$

SLOŽITOST ALGORITMŮ $\left\{ \begin{array}{l} \text{ČASOVÁ} \\ \text{PAMĚŤOVÁ} \end{array} \right.$

Příklad: spočítat $n!$

čas: $O(n)$
paměť: $O(1)$

```
r ← 1  
for i 1 to n: r ← r * i  
return r
```

RAM
random access
machine

trídění n čísel a_1, \dots, a_n

for $i := 1$ to n

$m := \min(a_1, \dots, a_n)$, $j := \text{index to } m$
přechod (i, m)

$$O((n-1) + (n-2) + \dots + 1) = O(n^2)$$

Merge Sort $\leadsto O(n \log n)$

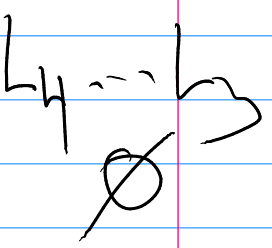
$T(n)$ Merge Sort ($a_1 \dots a_n$): if $n=1$ return a_1 else:
 $2 \cdot T(\frac{n}{2})$ $\left\{ \begin{array}{l} x_1 \dots x_{\frac{n}{2}} := \text{Merge Sort}(a_1 \dots a_{\frac{n}{2}}) \\ y_1 \dots y_{\frac{n}{2}} := \text{Merge Sort}(a_{\frac{n}{2}+1} \dots a_n) \end{array} \right. \quad \hookrightarrow = 2^i$

$O(n)$ $\leftarrow b_1 \dots b_n := \text{Merge}(x_1 \dots x_{\frac{n}{2}}, y_1 \dots y_{\frac{n}{2}})$
 return $b_1 \dots b_n$ beide L jetzt done \leadsto setzende LUP

```

Merge(L, P):
    i, j := 1
    for c := 1 to len(L) + len(P)
        if  $b_i < b_j$ :
             $c := L_i$ 
             $H = 1$ 
            if  $i > \text{len}(L)$ : break
        else:
             $c := P_j$ 
             $H = 1$ 
            if  $j > \text{len}(P)$ : break
    return  $b_1 \dots b_c | b_{c+1} \dots b_{\text{len}(L)} | P_j \dots P_{\text{len}(P)}$ 
    
```

$O(\text{len}(L) + \text{len}(P))$



$P = 134$ $L = 256$

$$\begin{aligned}
 T(n) &= 2 \cdot T\left(\frac{n}{2}\right) + n \\
 &= 2 \left(2 T\left(\frac{n}{4}\right) + \frac{n}{2} \right) + n \\
 &= 4 \cdot T\left(\frac{n}{4}\right) + 2n \\
 &= 8 \cdot T\left(\frac{n}{8}\right) + 3n = \dots = 2^i T\left(\frac{n}{2^i}\right) + in
 \end{aligned}$$

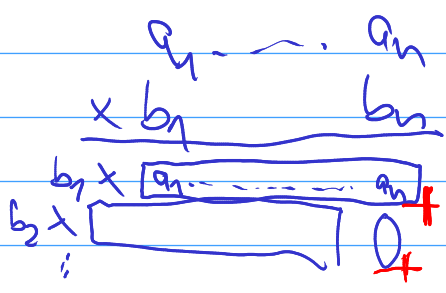
$i_s := \log_2 n$ kwocik

$$\Downarrow$$
$$\Rightarrow 2^{i_s} T(1) + i_s \cdot n =$$

$$n + \log n \cdot n \Rightarrow T(n) = \Theta(n \log n)$$

$a \times b$ násobení čísel (dlouhých)

~~$a_1 \dots a_n$~~
 ~~$b_1 \dots b_n$~~
 $O(k)$



$\Theta(n^2)$
 $\Theta(n^2)$
 $\Theta(n^2)$

násobení
 sčítání

$$X = \underbrace{x_1 x_2 \dots x_{n/2}}_A \underbrace{\dots x_n}_B$$

$$Y = \underbrace{y_1 y_2 \dots y_{n/2}}_C \underbrace{\dots y_n}_D$$

$$X = 10^{n/2} \cdot A + B$$

$$Y = 10^{n/2} \cdot C + D$$

$$XY = (10^{n/2} A + B)(10^{n/2} C + D)$$

$$= 10^n AC + 10^{n/2}(BC + AD) + BD$$

$$T(1) = 1$$

$$T(n) = 4T\left(\frac{n}{2}\right) + 2 \cdot n$$

$$= 4(4T\left(\frac{n}{4}\right) + 2\left(\frac{n}{2}\right)) + 2 \cdot n = 16T\left(\frac{n}{4}\right) + 3 \cdot 2n$$

$$64T\left(\frac{n}{8}\right) + 16 \cdot 2 \cdot \frac{n}{4} + 3 \cdot 2n = 64T\left(\frac{n}{8}\right) + \overset{(2^i - 1)}{7} \cdot 2 \cdot n$$

$$= \dots = 4^i T\left(\frac{n}{2^i}\right) + (2^i - 1) \Theta(n)$$

$$i = \log_2 n$$

$$\underbrace{2^{2/\log_2 n}}_{n^2}$$

$$n \cdot \Theta(n) = \Theta(n^2)$$

AC, BD

$$Z = (A+B)(C+D) = AC + \underbrace{BC+AD} + BD$$

$$Z - AC - BD = (BC+AD)$$

$$XY = (10^{n/2} A + B)(10^{n/2} C + D)$$

$$= 10^n \underbrace{AC}_{\boxed{AC}} + 10^{n/2} \underbrace{(BC+AD)}_{\boxed{BC+AD}} + \underbrace{BD}_{\boxed{BD}}$$

$\boxed{BC+AD}$

$$\Theta(n^{\log_2 3})$$

||

$$O(n^{1.6})$$

$$\underline{T(n)} = 3 \cdot T\left(\frac{n}{2}\right) + q \cdot n$$

$$= \dots = \dots$$

$$a=3, b=2, c=1$$

MASTER THEOREM:

Rekurencel $T(1)=1, T(n) = a \cdot T\left(\frac{n}{b}\right) + \Theta(n^c)$

podminky $a \geq 1, b > 1, c \geq 0$ má řešení

• $a = b^c \rightsquigarrow T(n) = \Theta(n^c \log n)$

• $a < b^c \rightsquigarrow T(n) = \Theta(n^c)$

• $a > b^c \rightsquigarrow T(n) = \Theta(n^{\log_b a})$