Start 11:35

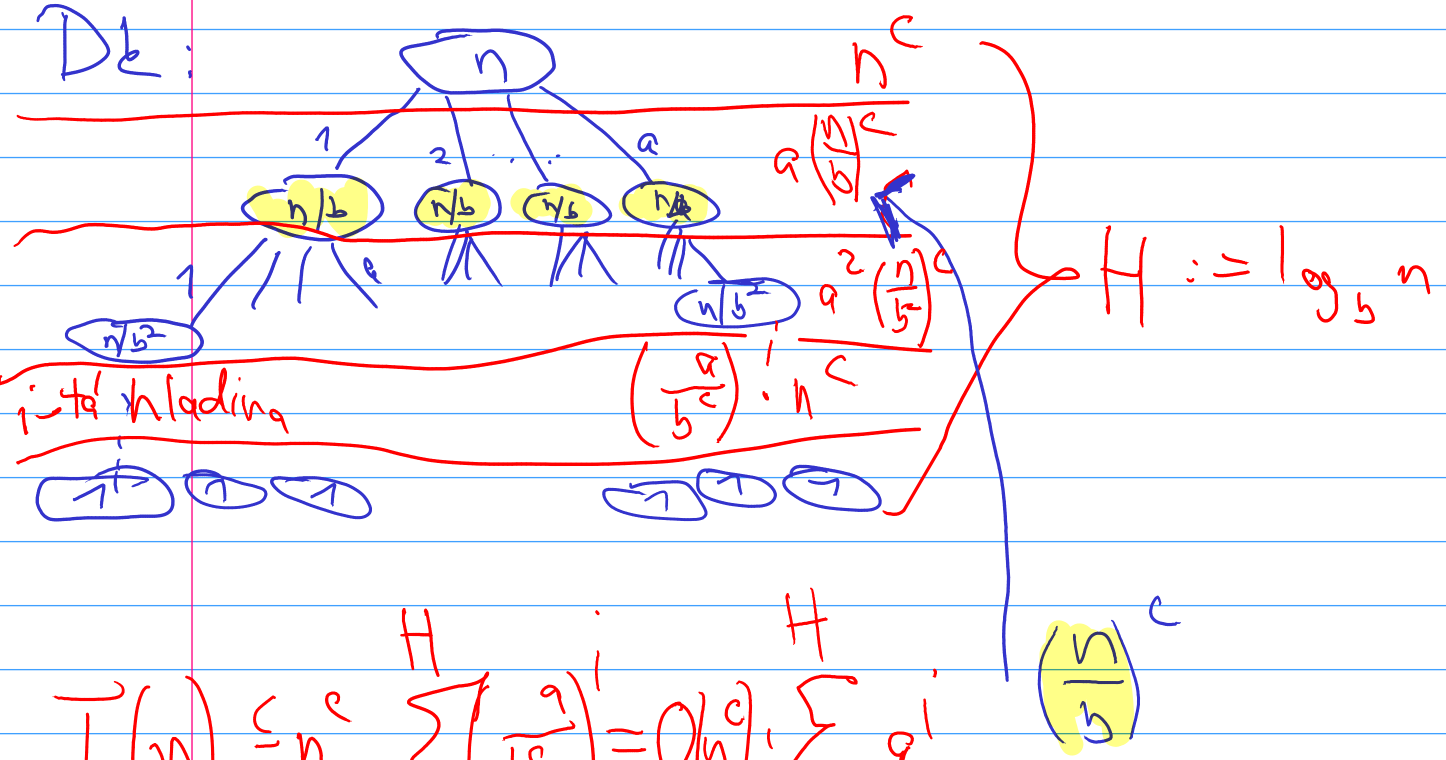
Věta (Master theorem): Rekurence tvaru

$$T(1)=1 \qquad T(n)=a\cdot T(n/b)+O(n^c)$$

má řešení v závislosti na $q := a/b^c$, práce na různých hladinách

- $q<1$ potom $T_n = O(n^c)$ nahoře > dole
- $q=1$ potom $T_n = O(n^c \log n)$ zhruba stejné
- $q>1$ potom $T_n = O(n^{\log_b a})$ dole > nahoře

Dk:



$$T(n) \le n^c \sum_{i=0}^{H} \left(\frac{a}{b^c}\right)^i = O(n^c)\cdot \sum_{i=0}^{H} q^i$$

$$q=1 \longrightarrow (H+1)\,O(n^c) \sim O(n^c \cdot \log n)$$

$$q<1 \rightsquigarrow O(n^c)\cdot \frac{1}{1-q} = O(n^c)$$

$$q>1 \rightsquigarrow O(n^c)\cdot \frac{q^{H+1}-1}{q-1} \le O(n^c)\cdot q^H = O(n^{\log_b a})$$

$$q^H = \left(\frac{a}{b^c}\right)^{\log_b n} = \frac{b^{\log_b a \cdot \log_b n}}{b^{c \cdot \log_b n}} = \frac{n^{\log_b a}}{n^c}$$

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median m $(a_1 \ldots \ldots a_n)$ $m \in \{a_1 \ldots a_n\}$

t.ž. $|a_i > m| \le n/2$

& $|a_i < m| \le n/2$

$a_1 < a_2 < \boxed{\ldots\ldots} \sim < a_n$

$m \in \{ a_{\lfloor \frac{n+1}{2} \rfloor}, a_{\lceil \frac{n+1}{2} \rceil} \}$ $\underbrace{O(n \log n)}_{\text{snadné}}$

ALG: k-tý nejmenší z $a_1 \ldots \ldots a_n$

VSTUP: (a_i), $k \in \{1, \ldots \ldots n\}$

Linear Select$((a_i), k)$:

- Pokud $n \le 5$: vyřeš hrubou silou
- Jinak rozdělme $\underbrace{(a_1 \ldots a_5)}_{P_1} \underbrace{(a_6 \ldots a_{10})}_{P_2} \ldots \underbrace{}_{P_i} \underbrace{(a_{n-4} \ldots a_n)}_{P_{\lceil n/5 \rceil}}$

$m_i := LS(P_i, 3)$ $\forall i$

q_1 L | P | q R | q_n

- $p := LS\left((m_i), \lceil n/10 \rceil\right)$

$L := \{q_i < p\}$, $\underbrace{P := \{q_i = p\}}$, $R := (q_i > p)$ $|L|+|P|+1$

Pokud $k \leq |L|$: vyjstup $LS(L, k)$

Jinak pokud $k \leq |L| + |P|$: vyjstup p

Jinak vyjstup $LS(R, k - |L| - |P|)$

$T(1) = 1$ $T(n) \leq T\left(\frac{n}{5}\right) + T(\max |R|, |L|) + \Theta(n)$

CLAIM: $\max \{|R|, |L|\} \leq \frac{7n}{10}$

$\Rightarrow$ V: $T(n) = \Theta(n)$

$T(1) = 1$, $T(n) = T\left(\frac{n}{5}\right) + T\left(\frac{7n}{10}\right) + A \cdot n$

DK (věty): indukcí dle n

BÚNO $A \geq 1$

$T(n) \leq 10 \cdot A \cdot n$ $n \leq 1$ ✓

$T(n) \leq 2An + 7An + An = 10An$

□

Dk (C(q, i, n))

$$m_1 \le m_2 \le \dots \le m_{n/10} \le \dots \le m_{n/5}$$

$$Z \le P$$

$$P \le \tilde{C}$$

$$|Z| = |\tilde{C}| \ge \frac{n}{10} \cdot 3 = 3n/10$$

$$Z \subseteq C \cup P \Rightarrow |R| \le 7n/10$$

$$\tilde{C} \subseteq P \cup R \Rightarrow |L| \le 7n/10$$

Násobení matic

$$\underset{n \times z}{X} \cdot \underset{z \times m}{Y} = \overset{n \times m}{\begin{pmatrix} m_{ij} \end{pmatrix}}$$

$$m_{ij} = \langle (X)_i, (Y^T)_j \rangle$$

$$\Theta(n^3)$$

Strassenův algoritmus $\Theta(n^{\log_2 7}) \approx n^{2.81}$

BÚNO $m = n = 2^k$

$$\begin{array}{c|cc} & n & 2^k - n \\ \hline z & X & 0 \\ 2^k - m & 0 & 0 \end{array} \qquad \begin{array}{c|cc} & z & \\ \hline m & Y & 0 \\ & 0 & 0 \end{array}$$

$k :=$ nejmenší že $2^k \geq m, n$

$$X \cdot Y = \begin{pmatrix} \overset{2^{k-1}}{A} & B \\ C & D \end{pmatrix} \cdot \begin{pmatrix} P & Q \\ R & S \end{pmatrix} = \begin{pmatrix} AP + BR & AQ + BS \\ CP + DR & CQ + DS \end{pmatrix}$$

Alg: Strassen

Rekurzivní $AP, BR, CP, DR, AQ, BS, CQ, DS$

součty $\Theta(n^2)$

$$T(n) = \cancel{8}\,7\, T\left(\frac{n}{2}\right) + \Theta(n^2)$$

$a = \cancel{8}\,7,\ b = 2,\ c = 2$

MThm

$$\Rightarrow \Theta\left(n^{\log_2 \cancel{8}\,7}\right) = \Theta\left(n^{2{,}81}\right)$$

TRIK!

$T_1 = (A+D)(P+S)$ $\quad$ $T_2 = (C+D)P$

$T_3 = A(Q-S)$ $\quad$ $T_4 = D(R-P)$

$T_5 = (A+B)S$ $\quad$ $T_6 = (C-A)(P+Q)$ $\quad$ $T_7 = (B-D)(R+S)$

$$X \cdot Y = \begin{pmatrix} T_1 + T_4 - T_5 + T_7 & T_3 + T_5 \\ T_2 + T_4 & T_1 - T_2 + T_3 + T_6 \end{pmatrix}$$