Instructions: The exam is 3 hours long and contains 6 questions. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers.**

Q1 Let G be the graph depicted in Figure 1.

a) Is G planar?

b) Find $\tau(G)$.

c) Show that $\chi(G) = 4$.
(*Hint:* To prove that $\chi(G) > 3$, show that in any 3-coloring of the graph $G \setminus v$, all three colors will appear on the neighborhood of v.)

Q2 Let G be the graph with weights $w : E(G) \to \mathbb{Z}_+$ depicted in Figure 2.

a) Find the min-cost spanning tree in G.

b) Find a shortest path spanning tree for the vertex A.

Q3 Let $k \geq 1$ be an integer, and let G be a connected k-regular bipartite graph. Show that G is 2-connected.

Q4 Let G be a planar graph. Show that if G does not contain any cycles of length five or less then $\chi(G) \leq 3$.

Q5 Let G be a 3-connected graph with $|V(G)| \geq 5$. Show that C_5 is a minor of G.

Q6 Let G be a simple graph. Suppose that there exists a clique $X \subseteq V(G)$ such that $V(G) - X$ is an independent set. Show that G is perfect without using the strong perfect graph theorem.

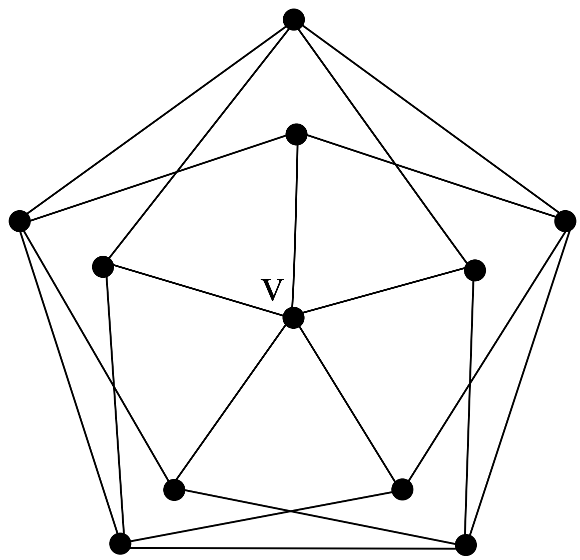

Figure 1: The graph in the question Q1.

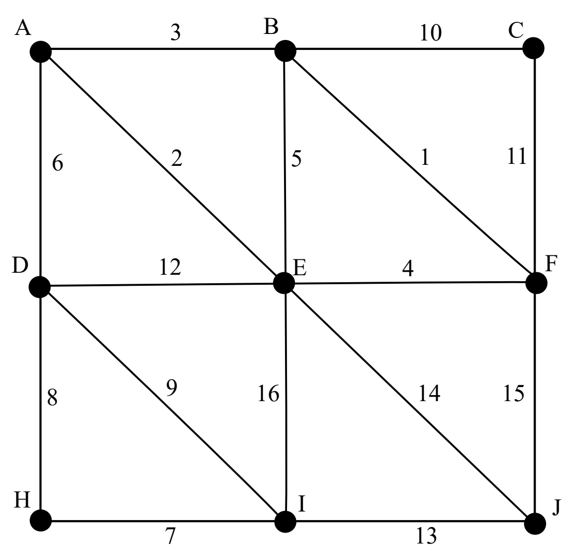

Figure 2: The graph in the question Q2.