

Instructions: The exam is 3 hours long and contains 6 questions. The total number of points is 100. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers!**

- Q1** Let G be the graph depicted in Figure 1.
- a) Is G planar? *(4 points)*
 - b) Find $\nu(G)$ and $\tau(G)$. *(4 points)*
 - c) Find $\chi(G)$. *(4 points)*
 - d) Find $\chi'(G)$. *(4 points)*
- Q2** Let $\vec{G} = (V, E)$ be the oriented graph with the two specific vertices s and t and with the capacities $c : E \rightarrow \mathbb{Z}_+$ depicted in Figure 2.
- a) Find a maximum flow from the vertex s to the vertex t . *(8 points)*
 - b) Find a minimum s, t -cut. *(8 points)*
- Q3** Let $G = (V, E)$ be the simple graph with weights $w : E \rightarrow \mathbb{Z}_+$ obtained from the oriented graph depicted in Figure 2 by replacing each oriented edge by a non-oriented one that has the same weight.
- a) Find a minimum-cost spanning tree in G . *(8 points)*
 - b) Does G have a unique minimum-cost spanning tree. *(8 points)*
- Q4** Let $k \geq 1$ be an integer, and let G be a connected $2k$ -regular graph. Show that G is 2-edge-connected. *(17 points)*
- Q5** Let G be a simple planar graph. Prove that if G contains no cycle of length five or less, then $\chi(G) \leq 3$. *(17 points)*
- Q6** Let K_4^- be the 4-vertex graph obtained from K_4 by removing one edge. How many non-isomorphic simple 2-connected graphs $G = (V, E)$ are there with $|V| = 1000$ such that G has no K_4^- -minor? *(18 points)*

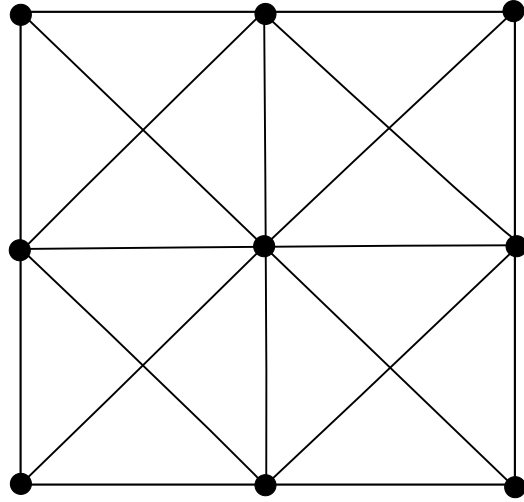


Figure 1: The graph in the question Q1.

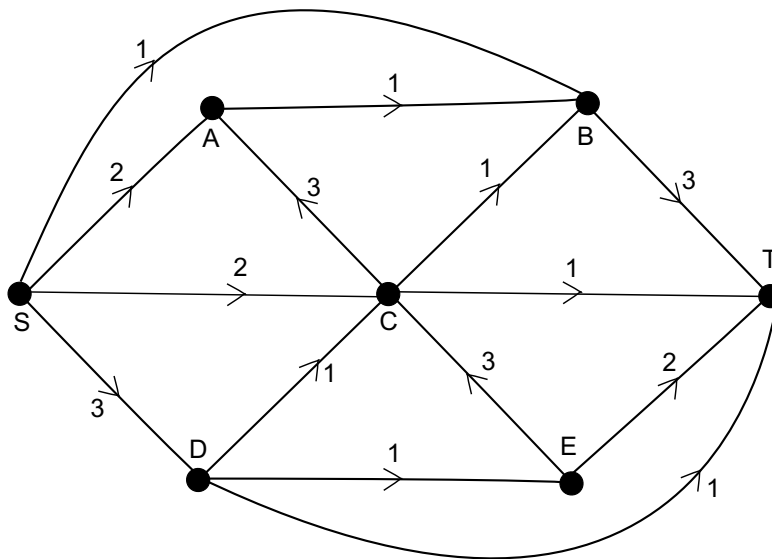


Figure 2: The oriented graph in the questions Q2 and Q3.