MATH 350: Graph Theory and Combinatorics. Fall 2017.
Assignment #11: Planar graphs

Due Thursday, November 30st, 8:30AM *Write your answers clearly. Justify all your answers.*

This is a warm-up question, do not submit your solution.

a) Find a simple 5-regular planar graph. Can you find a one with 12 vertices? *(0 points)*

b) Prove that every simple 5-regular planar graph must have at least 12 vertices. *(0 points)*

c) Find a simple planar graph G such that its dual is isomorphic to G. *(0 points)*

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1. Let G be a simple triangle-free planar graph. Without using the 4-Color Theorem, prove that $\chi(G) \leq 4$. *(2 points)*

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2. A simple graph G is called *outerplanar* if it can be drawn in the plane without any crossing in such a way that every vertex is incident with the infinite region.

Let $G = (V, E)$ be a connected outerplanar graph with $|V| \geq 3$.

a) Prove that G contains two vertices of degree at most 2. *(1 point)*

b) Without using the 4-Color Theorem, prove that $\chi(G) \leq 3$. *(1 points)*

c) Prove that a graph is outerplanar if and only if it contains no K_4-minor and no $K_{2,3}$-minor. *(2 points)*

3a) Let H be a simple graph with maximum degree at most 3. Show that every simple graph contains a subdivision of H if and only if it contains H as a minor. *(2 points)*

3b) Let G be a simple graph that contains K_5 as a minor. Prove that G contains a subdivision of K_5 or a subdivision of $K_{3,3}$. *(2 points)*

($\star$) **This is a challenge of the week question, do not submit your solution.**

Let G be a simple triangle-free planar graph. Prove that $\chi(G) \leq 3$. *(Compare this with the problem #1 !)*