Instructions: The exam is 3 hours long and contains 6 questions. The total number of points is 100. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers!**

Q1 Let G be the graph depicted in Figure 1.

a) Is G planar? *(4 points)*

b) Does G contain a Hamiltonian cycle? *(4 points)*

c) Find $\chi(G)$. *(4 points)*

d) Find $\chi'(G)$. *(4 points)*

Q2 Let $\overrightarrow{G} = (V, E)$ be the oriented graph with the two specific vertices s and t and with the capicities $c : E \to \mathbb{Z}_+$ depicted in Figure 2.

a) Find a maximum flow from the vertex s to the vertex t. *(8 points)*

b) Find a minimum s, t-cut. *(8 points)*

Q3 Let $G = (V, E)$ be the simple graph with weights $w : E \to \mathbb{Z}_+$ obtained from the oriented graph depicted in Figure 2 by replacing each oriented edge by a non-oriented one that has the same weight.

a) Find a minimum-cost spanning tree in G. *(8 points)*

b) Prove that G has a unique minimum-cost spanning tree. *(8 points)*

Q4 Let $G = (V, E)$ be a simple graph.

a) Prove that if G is 2-connected and $e, f \in E$ are two of its edges, then there exists a cycle in G containing both e and f. *(8 points)*

b) Is it true that if G is such that for all $e, f \in E$ there exists a cycle in G containing both e and f, then G is 2-connected? *(8 points)*

Q5 Let G be a simple planar graph. Without using the Four Color Theorem, Prove that if G does not contain a triangle, then $\chi(G) \leq 4$. *(18 points)*

Q6 How many non-isomorphic simple 2-connected graphs $G = (V, E)$ are there with $|V| = 1000$ such that G has no C_5-minor? *(18 points)*

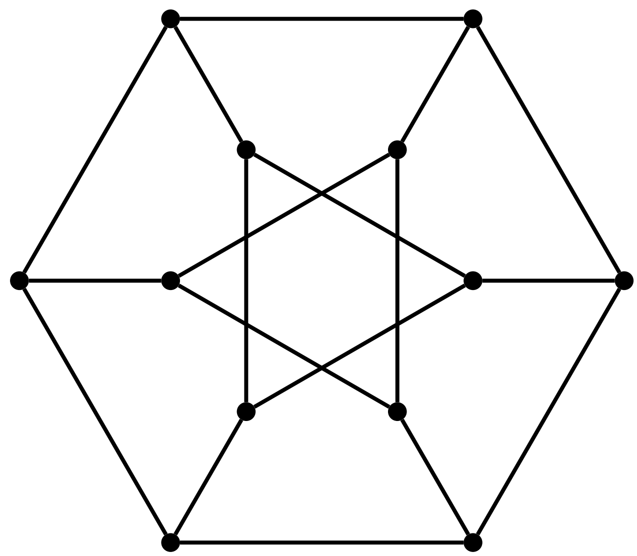

Figure 1: The graph in the question Q1.

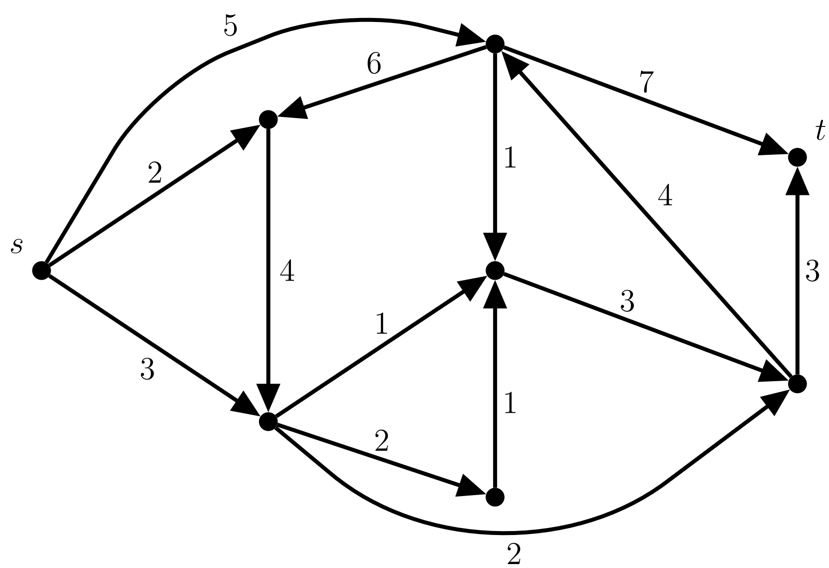

Figure 2: The oriented graph in the questions Q2 and Q3.