Instructions: The exam is 3 hours long and contains 6 questions. Write your answers clearly in the notebook provided. You may quote any result/theorem seen in the lectures without proving it. **Justify all your answers.**

Q1 Let G be the graph pictured on Figure 1.

    a) Is G planar?

    b) Is G perfect?

    c) Find $\chi(G)$.

    d) Find $\chi'(G)$.

Q2 Let G be the graph with weights $w : E(G) \to \mathbb{Z}_+$ pictured on Figure 2.

    a) Find the min-cost spanning tree in G.

    b) Find a shortest path spanning tree for the vertex S.

Q3 Let G be the digraph pictured on Figure 3 with the capacity $c(e)$ indicated for every edge $e \in E(G)$. Find the c-admissible (S, T)-flow in G of maximum total value.

Q4 Let G be a connected loopless graph with $|E(G)|$ even. Show that the line graph $L(G)$ of G has a perfect matching.

Q5 Let H be the loopless graph with $|V(H)| = 2$ and $|E(H)| = 3$. (That is, H consists of two vertices joined by three parallel edges.) Let G be a loopless graph with $\deg(v) \geq 4$ for all $v \in V(G)$. Show that G contains H as a minor.

Q6 Let G be a simple chordal graph with $|V(G)| \geq 10$ and $\omega(G) \leq 10$. Show that
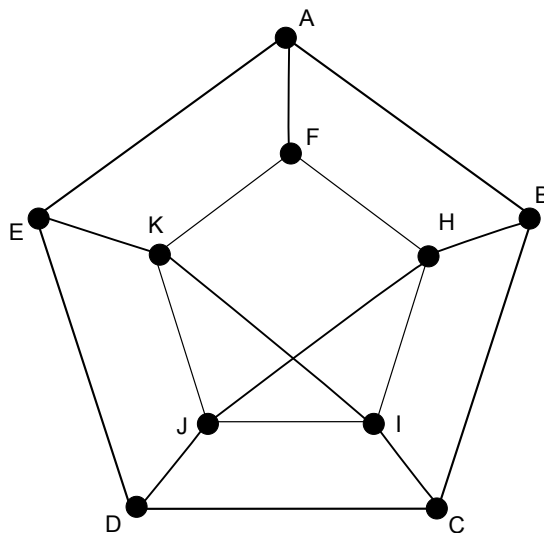$$|E(G)| \leq 9|V(G)| - 45.$$
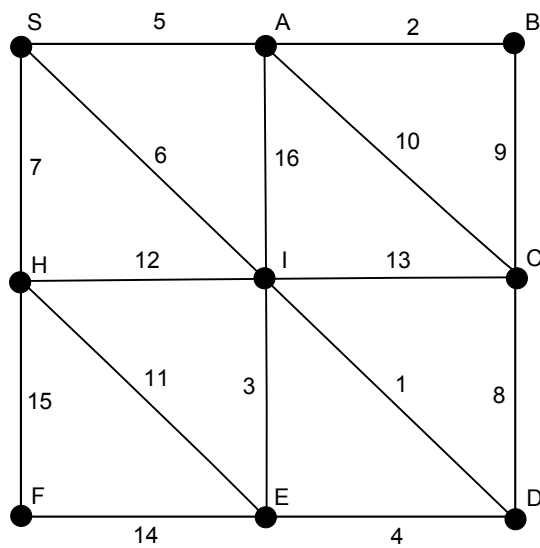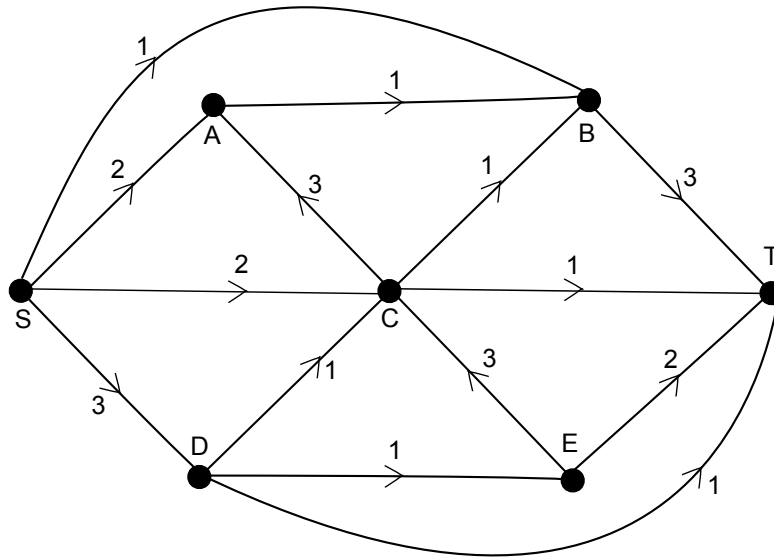
Figure 1: The graph in the question Q1.

Figure 2: The graph in the question Q2.

Figure 3: The graph in the question Q3.